Tracking performance characterization and improvement of a piezoactuated micropositioning system based on an empirical index

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ABSTRACT

Motion tracking is an important problem in micropositioning systems dedicated to ultra-precision robotic micromanipulation. This paper investigates the periodic motion tracking performance of a micropositioning system based on an empirical tracking performance index (TPI), which is proposed with the consideration of both tracking errors and tracking frequencies. To improve the TPI of a micropositioning system with piezoelectric actuation, a two-loop controller consisting of an $H_\infty$ robust controller and a disturbance observer is implemented. Experimental studies are conducted on an XY parallel micropositioning system for the motion tracking tests. Results demonstrate that a sole $H_\infty$ robust controller is insufficient to produce a satisfactory tracking performance with low-frequency control. In contrast, the two-loop controller endows the system with both a lower tracking error and a better TPI than the conventional robust control does. The experimental results not only validate the effectiveness of the proposed control method in improving tracking performance but also confirm the feasibility of the TPI in quantitatively characterizing the performance of a micropositioning system.

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1. Introduction

A micropositioning system is a crucial component for the robotic micromanipulation system dedicated to automated high-accuracy positioning and assembly in the field of micromanipulation [1]. In the literature, numerous micropositioning stages have been designed to deliver various types of motions [2–4]. Particularly, most stages employ piezoelectric actuators (PZTs) for the actuation. The reason lies in that PZTs can provide linear positioning with (sub)nanometer resolution, large blocking force, high stiffness, and rapid response characteristics. However, the hysteretic effects of the PZT, which arise from the voltage-driven strategy, introduce nonlinearities into the system. If they are not carefully treated, the nonlinearities will degrade the system's positioning accuracy drastically [5]. Thus, once a micropositioning stage is developed, its accuracy is dominantly dependent on the construction of an appropriate control scheme.

At the same time, motion tracking is an important capability for a micropositioning system. For example, in a scanning probe microscope, a periodic scanning motion between the probe and sample needs to be precisely followed by one axis to obtain the desired surface profile of the sample [6]. The performance of a micropositioning system for the motion tracking is traditionally characterized based upon its tracking errors in terms of the peak-to-peak error [7], maximum error [8,9], root mean square error [7,9], and relative maximum error [9], etc. Nevertheless, practical development issues of the system in terms of sampling frequencies of the sampled-data system and input rates of the reference signals are seldom considered for the performance measurement. Consequently, the traditional indices cannot differentiate two sampled-data micropositioning systems which achieve the identical tracking errors for the same reference input but with different sampling frequencies. Actually, a higher sampling frequency requires a larger cost for the hardware implementation of the system. From this economic point of view, to maintain a specified tracking error for a given reference input, the lower is the sampling frequency, the better is the system performance in terms of system costs. Therefore, a tracking performance index (TPI) with the consideration of both tracking errors and frequencies is desirable to characterize and compare micropositioning systems more reasonably. In the current research, such a TPI is proposed to quantitatively measure the system's tracking ability in a more comprehensive manner.

In order to overcome the hysteresis which dominates the nonlinearity and to achieve a fine tracking performance of a piezo-driven micropositioning system, different control approaches have been reported in the literature. Generally, the approaches can be classified into two categories in terms of the hysteresis model-based and hysteresis model-free methods. In the first category, a hysteresis model (e.g., the Preisach model) is generated and used to construct an inverse-based feedforward compensator [10]. It has been shown that the inverse-based
compensation can achieve a good tracking, whereas the result is very sensitive to the model accuracy [6]. Thus, another approach is to combine the feedforward with a feedback control to suppress the hysteresis as well as creep and drift effects in the PZT [11–14]. Usually, the generation of a hysteresis model is a complex procedure, which implies a time consuming work for the control design process. On the contrary, the major advantage of the second category of control methods lies in that no hysteresis model is required. Alternatively, the unmodeled hysteresis is considered as an uncertainty or a disturbance [15] to the nominal system, and a robust control is then constructed to tolerate it. For instance, the sliding mode control [16,17], $H\infty$ robust control [8,18], fuzzy logic control [19], and neural network-based control [20,21] have been presented in recent works. Besides, some other control approaches [22–25] have been reported as well.

In this research, the hysteresis model-free method is adopted for the purpose of rapid prototyping of the controller. Specifically, an improved robust control, i.e., a two-loop control framework employing an $H\infty$ robust control combined with a disturbance observer (DOB), is presented to suppress the unmodeled hysteresis and other nonlinearities in a piezo-driven micropositioning system. It is noticed that a similar controller has been reported in [26] to compensate the variant friction force in an over-constrained parallel kinematic machine. In the current work, it will be experimentally demonstrated that the two-loop control strategy enables a better tracking performance of a piezo-driven micropositioning system than the single $H\infty$ control does. Moreover, it will be shown that the presented controller has a simple structure and is easy to implement.

In the rest of the paper, the new TPI is proposed in Section 2 along with some existing works characterized and compared for illustrations. The system description of a test-bed employed in the research and plant model identification are reported in Section 3. Based on the identified model, an $H\infty$ robust control is then designed in Section 4, where the performance of the micropositioning system with the controller is characterized. To improve the system tracking ability, an integrated two-loop controller is proposed in Section 5 with experimental verification carried out. Afterwards, Section 6 outlines the current research limitations and future works and Section 7 summarizes the paper.

2. Tracking performance index

2.1. Limitations of traditional indices

Traditionally, the motion tracking control performance of a micropositioning system is characterized based upon the tracking errors. For example, the maximum, mean, standard derivation, root mean square, and relative maximum of tracking errors are the most used indices to measure the tracking performance. The relative maximum error (RME) can be defined as the ratio of the maximum absolute tracking error with respect to the peak-to-peak amplitude of the input signal. Generally, the higher is the sampling frequency, the smaller are the tracking errors. In addition, the smaller are the errors, the better is the system tracking ability. Hence, it is commonly recognized that the higher is the sampling frequency, the better is the system performance.

However, since the traditional performance measures only emphasize on the tracking errors yet do not consider the sampling rate or frequency, the traditional indices cannot differentiate two sampled-data micropositioning systems which achieve the identical tracking errors with the same reference input signal but different sampling frequencies. Actually, a higher sampling frequency requires a larger cost for the hardware implementation. If two systems achieve the identical tracking errors for the same input signal, the system with a lower sampling rate requires a smaller hardware cost in terms of A/D sampling. From this economic point of view, to maintain a specified tracking error for a given reference input signal, the lower is the sampling frequency, the better is the system tracking performance. Hence, a more compensative performance index with the consideration of both tracking errors and frequencies is required to compare the systems more fairly.

2.2. Proposal of an empirical TPI

For the sake of characterizing a micropositioning system in a more comprehensive manner, an empirical TPI is proposed in this paper as an empirical index to characterize the system more reasonably. Specifically, with the four parameters $e_{\text{rm}}, A_{\text{r}}, f_{\text{r}}$, and $f$ describing the maximum tracking error, amplitude of reference input, rate of reference input, and closed-loop sampling frequency, respectively, the TPI (ξ) is defined in such a form

$$\xi = \frac{e_{\text{rm}}}{A_{\text{r}} f_{\text{r}}}$$  \hspace{1cm} (1)

where the ratio $e_{\text{rm}}/A_{\text{r}}$ denotes an RME, i.e., the ratio of the maximum tracking error relative to the amplitude of reference input signal. In addition, the ratio $f_{\text{r}}/f$ can be considered as the sampling rate utility ratio of the system.

For a scanning probe microscope, the input rate $f_{\text{r}}$ of the reference input signal influences the speed of the scanning process, and a larger input signal rate results in a higher scanning speed. Hence, if two systems have the identical sampling frequencies, the larger is the rate of the reference input signal that can be accurately tracked, the better is the tracking ability of the system, since a larger $f_{\text{r}}$ means less time required to track the same trajectory. Therefore, the larger is the ratio $f_{\text{r}}/f$, the higher is the efficiency of the system.

As a combination of the above two individual ratios, the proposed TPI takes into account not only the tracking error but also the cost (both computational and economical) of the system. Moreover, the smaller is the TPI, the better is the tracking performance of a micropositioning system. This concept can be understood by reconsidering an example scenario. Two micropositioning systems are supposed to get the identical RME for the same reference input signal with different sampling frequencies. A lower sampling frequency means a smaller (computational and economical) cost requirement for the hardware. Therefore, the smaller is the TPI, the better is the system performance.

It is noticeable that the TPI is proposed here to characterize and compare micropositioning systems for the position tracking of reference input of periodic signals. If the input is a DC signal such as the step input, the traditional indices only based on the tracking errors are sufficient to characterize the system. Thus, the TPI can be used as a supplementary measure in addition to traditional indices.

For illustrations, both RME and TPI values of some existing PZT or piezo-driven micro/nano-positioning systems for the motion tracking of sinusoidal reference inputs with various control strategies are elaborated in Table 1. Although a great number of literature deals with periodic motion tracking control of precise positioning systems, most of them do not provide full details on the four parameters used to calculate their RME or TPI. Concerning each work listed in Table 1, in case that different RME or TPI values are calculated for the tracking of different trajectories, the best ones are selected to characterize the system. From the table, we can observe that the traditional and proposed indices rank a micropositioning system in different ways. Among these works, the number 4 achieves the minimum RME of 0.16%. However, it is obtained using hardware with a higher cost
compared to other works as reflected by its relatively larger TPI value of 5.76.

For the same or different systems, the values of the RME or TPI would vary, depending on the frequency of input signals to the system. This implies that the comparison based on RME or TPI makes sense only for the same input frequency. In the following discussions, the control system for a recently developed micro-positioning platform [28] is constructed to improve its tracking capability, i.e., to achieve a smaller value of TPI.

3. System description and identification

The schematic diagram of the test-bed employed in this research, an XY parallel-kinematic micropositioning stage, is shown in Fig. 1. The stage is constructed by four identical PP (P stands for prismatic joint) limbs and actuated with two piezoelectric actuators (PZTs) through two displacement amplifiers. It is known that the PZT cannot bear transverse loads due to the risk of damage, which usually arises from the influences of another PZT. The adopted displacement amplifier acts as an ideal P joint and possesses a large ratio of stiffness in transverse direction to that in working direction. Hence, the amplifier also acts as a decoupler with the roles of transmitting axial force of actuator and preventing the actuator from suffering undesired transverse motion and loads as well. Thus, the two actuators are well isolated and protected. Moreover, the ideal translation provided by compound parallelogram flexures allows the generation of decoupled output motion of the stage. Since the employment of only two PP limbs is sufficient to obtain an XY translation, four limbs are used to construct a symmetric structure to enhance the accuracy performance.

More details about the stage structure and working principle can be found in [29]. For the reason of completeness, its working principle is briefly described here. Referring to Fig. 1, we can observe that, once the stage is driven by PZT 1 with an input displacement $q_1$, the displacement is amplified by the amplifier 1 and then transmitted to the output platform of the stage. This leads to an output displacement $x$ along the $x$-axis. Similarly, an input motion $q_2$ produces a $y$-axis output displacement $y$ of the stage. Since the stage is well decoupled, the two axial motions can be treated independently.

3.1. Experimental setup

The experimental setup for the micropositioning system is graphically shown in Fig. 2. The monolithic XY stage is fabricated from a piece of light material Al7075-T651. Two 20 μm- stroke PZT (model PAS020 produced by Thorlabs, Inc.) are adopted to drive the stage, and the PZTs are actuated through a two-axis piezo-amplifier and driver (BPC002 from the Thorlabs) with a voltage ranging from 0 to 75 V. By mounting two blocks with smooth surface finish on the output platform, the displacements of the end-effector are measured by two laser displacement sensors (Microtrak II, head model: LTC-025-02, from MTI Instruments, Inc.). The analog voltage outputs (within ± 5 V) of the two sensors are connected to a peripheral component interconnect (PCI)-based data acquisition (DAQ) board (PCI-6143 with 16-bit A/D converters, from NI Corp.) through a shielded I/O connector block (SCB-68 from the NI) with noise rejection. The digital outputs of the DAQ board are then read by a personal computer (PC) through the PCI local bus.

Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Control method</th>
<th>RME (%)</th>
<th>TPI</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fuzzy decentralized control</td>
<td>1.83</td>
<td>1.17</td>
<td>[19]</td>
</tr>
<tr>
<td>2</td>
<td>Inverse model feedforward + PI</td>
<td>1.88</td>
<td>1.88</td>
<td>[12]</td>
</tr>
<tr>
<td>3</td>
<td>Inverse MPP feedforward</td>
<td>3.68</td>
<td>2.48</td>
<td>[10]</td>
</tr>
<tr>
<td>4</td>
<td>Inverse MPI feedforward + SMC</td>
<td>0.16</td>
<td>5.76</td>
<td>[13]</td>
</tr>
<tr>
<td>5</td>
<td>Adaptive SMC</td>
<td>1.67</td>
<td>6.84</td>
<td>[16]</td>
</tr>
<tr>
<td>6</td>
<td>Recursive control</td>
<td>10.00</td>
<td>10.00</td>
<td>[5]</td>
</tr>
<tr>
<td>7</td>
<td>PID* + feedback linearization + RC</td>
<td>3.57</td>
<td>10.71</td>
<td>[23]</td>
</tr>
<tr>
<td>8</td>
<td>SMC</td>
<td>0.67</td>
<td>13.33</td>
<td>[17]</td>
</tr>
<tr>
<td>9</td>
<td>Inverse Preisach feedforward + PD</td>
<td>1.88</td>
<td>18.75</td>
<td>[14]</td>
</tr>
<tr>
<td>10</td>
<td>Inverse model feedforward</td>
<td>2.00</td>
<td>20.00</td>
<td>[27]</td>
</tr>
</tbody>
</table>

* Proportional–integral control.
* Modified Prandtl–Ishlinskii model.
* Sliding model control.
* Proportional–integral–derivative control.
* Repetitive control.
* Proportional–derivative control.
Since the sensitivity of the laser sensor is 2.5 mm/10 V and the maximum value of 16-bit digital signal corresponds to 10 V, the resolution of the displacement detecting system can be derived as 0.038 μm. However, due to a not negligible level of the noise, the resolution of the sensor is claimed as 0.1 μm by the manufacturer. A preliminary open-loop test shows that the micropositioning platform has a workspace around the area of $117 \, \mu m \times 117 \, \mu m$ with a maximal cross-talk of 1.5% between the two axes, which verifies the well-decoupling property of the XY stage.

Based on the hardware available, a PC-based control will be implemented in this research. According to the discrete-time control theory for sampled-data systems, more than 20 times of the closed-loop system bandwidth is preferred for the sampling frequency of discrete control [30, Chapter 11]. Because a maximal closed-loop sampling rate of 50 Hz can be achieved by the hardware, a maximal bandwidth of 2.5 Hz is expected from the control system. It is noticeable that much higher sampling rate and system bandwidth can be obtained if a digital signal processor (DSP)-based control is implemented.

3.2. Plant model identification

The system plant consists of the components of piezoelectric actuators, PZTs, XY stage, and displacement sensors. Since it is difficult to obtain an accurate physical model of the whole system, a linear model of the plant is identified experimentally about a specific operating point. The identified model is called a nominal model of the system, and the uncertainty to the nominal model is expected to be compensated by designing a robust controller.

The time domain method is adopted for the model identification since it is more reliable than the frequency domain based sine-sweep method [8]. Due to the two motions of the XY stage are well-decoupled, plant model in the directions of $x$- and $y$-axes can be identified independently. Thus, when the stage is driven by one PZT, the other PZT remains undriven. Owing to the rich frequency spectrum, a pseudo-random binary sequence (PRBS) signal is used as the input signal to PZT. Because of the piezoelectric actuation, hysteresis effects will become significant once the input signal has a large amplitude (compared to the input voltage range of 75 V). The plant model can be assumed to be linear over a short operation range around an operating point [18]. Thus, to obtain a linear model of the system, a low-pass filter with a second-order roll-off of 40 dB/decade. This technique is chosen to obtain the transfer function of the plant using MATLAB, which gives a third-order model shown below:

$$ G_{sL}(s) = \frac{-0.4496s^2 + 7957s + 1.455 \times 10^5}{s^3 + 117.4s^2 + 8502s + 1.168 \times 10^5} $$

which represents a nominal model of the plant $G_s$.

The bode diagram of the plant model transfer function $G_{sL}$ is plotted in Fig. 4, which indicates that the system looks like a low-pass filter with a second-order roll-off of 40 dB/decade. This phenomenon does not concord with those of some other micro-positioners reported in the literature, which have a dominant lightly damped resonant mode at relatively low frequencies (in hundreds of Hz) [6,8,18,24]. Actually, the resonant peak is removed here by the adoption of the low sampling frequency (50 Hz) [28]. Hence, the derived plant model can only represent the system in low frequency range (less than Nyquist frequency, 25 Hz), since it does not reflect the resonant mode of the system in higher frequencies. So, the identified plant model transfer function is only sufficient for the purpose of low-frequency control design.

Fig. 3(b) reveals that the simulated model output matches the experimental data with a fit value of 84.6%, which indicates the match rate between the simulated model output and measured output. It is found that the fit value relies on the noise level of the experimental data, that is the reason why the collected output data are averaged instead of using an individual set of data for the model identification purpose. Although the fit rate can be further increased by averaging more sets of data collection experimental outputs, there always exists a mismatch between the model
output and measured output. Moreover, the identified model varies with respect to the changes of operating point over the workspace range of the device.

These uncertainties in terms of model incompleteness and model variation are the essentials of piezo-driven systems attributed to hysteretic and other nonlinearities. To compensate for the nonlinearity, an $H_\infty$ robust control scheme is designed in the subsequent section.

4. Robust controller design and verification

With comparison to other feedback control approaches, the mixed-sensitivity $H_\infty$ robust control exhibits the advantage of combining the performance and robustness requirements in one control design scheme. The achievement of the performance and robustness for $H_\infty$ controller heavily depends on the weighting functions selection.

4.1. Robust controller design

The block diagram of the closed-loop $H_\infty$ robust control is shown in Fig. 5, where $W_1(s)$, $W_2(s)$, and $W_3(s)$ represent the weighting functions. Additionally, $z_1$, $z_2$, and $z_3$ are weighted signals. The goal of $H_\infty$ controller design is to minimize the transfer functions of these weighted signals. The sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ of the closed-loop system can be respectively obtained as

$$S = \frac{1}{1 + G_sK}, \quad T = \frac{G_sK}{1 + G_sK},$$

where the argument ($s$) is omitted for the sake of concise representation. Additionally, $G_s$ is the plant model and $K$ denotes the $H_\infty$ controller to be designed.

In order to design the controller, three weighting functions ($W_1$, $W_2$, and $W_3$) need to be first determined to shape the sensitivity function $S$, control transfer function $KS$, and complementary sensitivity function $T$, respectively. With reference to Fig. 5, we can deduce that $S$ denotes the transfer function between the reference input $x_d$ and tracking error $e$, $KS$ relates $x_d$ to the control input $u$, and $T$ relates $x_d$ to the sensor output $x$ as well as the sensor noise $n$ to sensor output $x$, respectively.

Based on the small gain theorem, the controller $K$ can be synthesized by minimizing the transfer functions of the weighted signals ($z_1$, $z_2$, and $z_3$), i.e.,

$$\|N\|_{\infty} = \sup_{\omega \in \mathbb{R}} |N(j\omega)| \leq 1, \quad N = \begin{bmatrix} W_1S & W_2KS & W_3T \end{bmatrix},$$

where $\| \cdot \|_{\infty}$ denotes the $H_\infty$ norm to measure the size of a transfer function, which is considered as the maximum singular value $|N(j\omega)|$ of the transfer function with the argument $\omega$ denoting the frequency. Such a controller guarantees that $\|W_1S\|_{\infty} \leq 1$, $\|W_2KS\|_{\infty} \leq 1$, and $\|W_3T\|_{\infty} \leq 1$. It follows that the inverse of the three weighting functions provide bounds on the corresponding transfer functions. Therefore, the key technique of designing an $H_\infty$ robust controller lies in the selection of the weighting functions.

Specifically, the weighting function $W_1$ is desired to exhibit high gains at low frequencies and low gains at high frequencies. This ensures that the sensitivity function is small at low frequencies so as to guarantee a small tracking error within the bandwidth of interest. In contrast, the weighting function $W_3$ is chosen such that it has low gains at low frequencies and high gains at high frequencies. This strategy is taken to ensure that the complementary sensitivity function rolls off at high frequencies to attenuate the sensor noise and hence to produce a better resolution. Besides, the weighting function $W_2$ is usually chosen as a constant to make the control input of PZT stay within the saturation limits.

In this research, the weighting function $W_1$ (see Fig. 6) is designed to be a first-order low-pass filter in the form

$$W_1(s) = \frac{s + \omega_B}{s + \omega_B + M}.$$  

According to the achievable closed-loop bandwidth with the current sampling rate, $\omega_B = 9.42 \text{ rad/s}$ (i.e., $1.5 \text{ Hz}$) in Eq. (6) is selected which represents the approximate bandwidth cross over the magnitude of 1. Additionally, the magnitude $N=0.05$ is chosen as a bound at low frequencies, and $M=3$ is selected as a bound at high frequencies. The function $W_1$ is designed to make the sensitivity function $S$ have a gain less than $-26 \text{ dB}$ at low frequencies with a bandwidth of $1.8 \text{ Hz}$ (see Fig. 7(a)) and thereby insensitive to the low frequency variations in the nominal model.

The weighting function $W_1$ (see Fig. 6) is selected as a first-order high-pass filter transfer function:

$$W_3(s) = \frac{s + \omega_B}{s + \omega_B + M}.$$  

where $\omega_B = 12.57 \text{ rad/s}$, $N=0.05$, and $M=3$ are chosen. This function is designed to enable the complementary sensitivity function $T$ to have a gain less than $-26 \text{ dB}$ at high frequencies with a roll-off slope of $-12 \text{ dB/decade}$ (see Fig. 7(b)), which is utilized to attenuate the noise higher than $2 \text{ Hz}$. 

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**Fig. 4.** Bode diagram of the identified plant model transfer function $G_{in}$. The resonant mode is not exhibited due to the adopted low sampling frequency.

**Fig. 5.** Block diagram of the mixed-sensitivity $H_\infty$ robust control system, which is equivalent to a standard two-part robust control system.
Besides, the weighting function for control input scaling is chosen as a constant $W_2 = 0.1$ so as to limit the control signal within the saturation limits of 75 V.

In view of the selected weighting functions, an optimal $H_\infty$ robust controller is designed based on the $\gamma$-iteration method (using the function “hinfopt” in Robust Control Toolbox of MATLAB). The obtained controller is a fifth-order model. For the convenience of implementation, the order of the controller is reduced by examining the Hankel singular values of the system as shown in Fig. 8, which indicates that the first three states dominate the energy of the system. The Bode frequency response diagrams of the open-loop system with different reduced orders of the controller (see Fig. 9) reveal that the third-order model approximates the original controller more accurately. Hence, the last two states are removed, and the reduced order controller is described below:

$$K(s) = \frac{8068s^2 - 6.578 \times 10^5 s + 2.369 \times 10^7}{s^3 + 2185s^2 + 2.554 \times 10^8 s + 1.324 \times 10^9},$$

(8)

which is then converted to a discretized-time mode with a sampling time interval of 0.02 s. With the reduced third-order controller, a gain margin of 11.3 dB and a phase margin of 68.6° can be observed from Fig. 9, which confirms that the control system is stable enough.

It is noticeable that the analog controller designed above is discretized directly by the conventional method of zero-order hold. It is verified that the selected sampling time does not degrade the performance of the analog controller. For an optimal discretization of the analog controller with the consideration of the discretization effect, the technique proposed in [31] can be employed, which is a remarkable method especially when the sampling rates are relatively low due to hardware constraints.

Furthermore, the comparisons of singular values plotted in Figs. 7(a) and (b) show that the sensitivity and complementary functions are covered by the inverse of the weighting functions $W_1$ and $W_3$, respectively, which ensure that the tracking performance (in terms of tracking error) as well as bandwidth and resolution performance are maintained. Moreover, a small...
The tracking results are shown in Fig. 10, which indicates that the sampling frequency of 50 Hz, it can be derived that the micropositioning platform. The platform is commanded to track a sinusoidal motion tracking results with the \( H_{\infty} \) controller enables an RME of 2.30% and a TPI of 11.50 for the peak 0.15 dB of the \(|S|_\infty\) implies a good robustness to model uncertainties.

4.2. Experimental verification

With the \( H_{\infty} \) controller designed above, an experimental study is carried out to test the motion tracking performance of the XY micropositioning platform. The platform is commanded to track a 0.1-Hz sinusoidal reference input with an amplitude of 110 \( \mu \)m. The tracking results are shown in Fig. 10, which indicates that the tracking error ranges between 2.53 and 2.40 \( \mu \)m. In view of the sampling frequency of 50 Hz, it can be derived that the \( H_{\infty} \) controller enables an RME of 2.30% and a TPI of 11.50 for the micropositioning system. With comparison to the index values tabulated in Table 1, it is realized that the XY micropositioning system with the \( H_{\infty} \) robust controller has a worse performance than most of the existing works, which is not sufficient for practical applications.

Although this experimental result does not concord with the conclusions reported in the previous works [8] which demonstrates that the nonlinearity of a piezo-driven nanopositioner can be completely compensated by the design of a robust controller, it accords with the result presented in [32] which illustrates that the robust design treating the uncompensated hysteresis and nonlinearities as unmodeled disturbances cannot achieve a good design specification. To accomplish the control design goal, a robust control incorporating an inverse-based compensator is suggested in [32]. In this research, instead of constructing an inverse hysteresis model to compensate for the hysteresis, the unmodeled hysteresis is treated as a disturbance and suppressed by a disturbance observer (DOB). The robust control combining with a DOB is recommended to compensate for the unmodeled hysteresis and other nonlinearities of a piezo-driven system. The realization of such a concept is illustrated in the subsequent section.

5. Robust control combined with disturbance observer and verification

The block diagram of the proposed two-loop robust control incorporating a DOB is depicted in Fig. 11, which exhibits a simple control structure. The disturbance \( d \) represents the unmodeled nonlinearity in the system. It has been shown that the outer loop robust controller and inner loop observer can be designed independently [26]. Actually, the DOB is used to improve the tracking precision of the system based on the stability provided by the controller \( K \). As a continuation of the design of an \( H_{\infty} \) robust controller in the previous section, the design of a DOB is carried out below.

5.1. DOB design

From Fig. 11, it can be observed that the DOB consists of the inverse of the nominal model \( G_{\text{sys}}^{-1} \) and a filter \( Q \). The DOB estimates the disturbance \( d \) by subtracting the estimated voltage input obtained by the inverse of the nominal plant model from the control input \( u_2 \). The filter is adopted to make the observer proper and realizable, which means that the relative degree of the filter should be no less than that of the plant model. Excluding the outer controller \( K \), the sensitivity function and complementary sensitivity function of the system can be derived approximately as [33]

\[
S_0 = 1 - Q, \tag{9}
\]

\[
T_0 = Q. \tag{10}
\]

It can be deduced that the sensitivity function \( S_0 \) relates the disturbance \( d \) to the control input \( v \), and the complementary sensitivity function \( T_0 \) relates the sensor noise \( n \) to the sensor output \( x \). Therefore, \( Q \) should be designed as a low-pass filter to make \( T_0 \) attenuate the sensor noise at high frequencies and to make \( S_0 \) suppress the disturbance \( d \) at low frequencies. Thus, increasing the order and bandwidth of the filter leads to the enhancement of the tracking performance of the system. However, excessive order and bandwidth of \( Q \) can result in instability caused by the plant uncertainty in higher frequencies. Therefore, once the controller \( K \) is constructed, a tradeoff between the performance and stability is needed to design the filter \( Q \).

In this research, considering that the nominal model has one relative degree as implied in Eq. (2), a third-order binomial filter with two relative degrees is employed as follows [34]:

\[
Q(s) = \frac{3(\tau s + 1)}{(\tau s)^2 + 3(\tau s) + 3(\tau s) + 1}, \tag{11}
\]

where \( \tau \) denotes the time constant of the filter. In order to reject the disturbance over a wide range of frequencies, a small value of \( \tau \) is desired. For a continuous-time system, the plant can always be stabilized by shrinking \( \tau \) if the system is of minimum phase [35]. However, in case of discrete-time system, too small \( \tau \) will induce destabilizing phenomenon of the system due to the discrete-time behavior of the sampled-data system. With the selection of \( \tau = 0.2 \), the magnitudes for the frequency responses of \( S_0 \) and \( T_0 \) are plotted in Fig. 12, which indicate the filter \( Q \) has a bandwidth of 1.31 Hz.
5.2. Integrated controller implementation

With the designed DOB, the total control input can be obtained by referring to Fig. 11 as follows:

\[ V(s) = \frac{1}{1-Q(s)} U(s) - \frac{C^{-1}_m(s)Q(s)}{1-Q(s)} X(s), \]  

(12)

where \( V(s) \), \( U(s) \), and \( X(s) \) represent the Laplace transforms of the total control input \( v(t) \), robust control input \( u(t) \), and sensor output \( x(t) \), respectively. The above continuous-time transfer function is converted into a discrete-time function with the sampling time interval of 0.02 s, which is then used to obtain the sampled control command \( v(t) \) in the time domain for the digital control.

5.3. Experimental verification

Using the \( H_\infty \) robust controller combined with DOB designed above, experimental tracking of the same sinusoidal reference input as described in Section 4.2 is conducted to test the performance of the micropositioning system. The tracking results are shown in Fig. 13. It is seen that the tracking errors lie in the range between −0.83 and 1.12 \( \mu \)m. Thus, the integrated controller allows an RME of 1.02% and a TPI of 5.09 for the micropositioning system.

With comparison to the \( H_\infty \) robust control results obtained in Section 4, it is obvious that the \( H_\infty \) robust control combined with DOB renders the system a significantly improved performance since both RME and TPI have been reduced by more than two times. Moreover, compared to the indices listed in Table 1, the XY micropositioning system with the two-loop controller has both a relatively smaller RME and a lower TPI than majority of existing works in the literature.

6. Limitations and future works

This research attempts to introduce a fairer index to characterize and compare micropositioning systems. An empirical TPI with the consideration of four parameters are proposed and illustrated through experimental studies. Due to the low sampling rate of the implemented PC-based control, the bandwidth of the closed-loop system is limited. Hence, the validation of the proposed index and controller is only accomplished for the low-frequency operation. A verification in high-frequency range is planned in the future works with the realization of a DSP-based control.

Moreover, the proposed index is far from mature. For a more sufficient characterization and comparison of the systems, other important aspects of the systems in terms of sensitivity of sensors, workspace of positioning systems, cost generated by the system update rate, dead-time and system delay, control bandwidth, positioning bandwidth, and calculation complexity of controllers, etc., are also expected to be taken into consideration. It should be noticed that the calculation complexity not only depends on the system update rate, but also on the employed control approach.

Besides, from another point of view, the higher sampling rate is favorable to choose higher control gains in order to make the system stiffer and hence overcome external disturbances. This is the reason why as reported in the robot position control literature in 1990s, the higher is the sampling frequency, the better is the command tracking ability. Hence, the future performance index should also include the relationships between the robustness and sampling frequency of the systems to characterize and compare the system tracking abilities at the presence of external disturbances.

7. Conclusions

The proposals of a tracking performance index for a more reasonable performance characterization and a two-loop control scheme for the performance improvement of micropositioning systems are the major contributions of this research. Comparing to traditional indices which measure the tracking performance of positioning systems based on tracking errors only, the introduced index is intended to characterize the systems more fairly by an integrated consideration of both tracking errors and the cost of system hardware, specifically the A/D sampling. Based on the system plant model identification and modern control theory, a two-loop control incorporating an \( H_\infty \) robust control and a disturbance observer is constructed to compensate for the nonlinearity presented in the piezo-driven system. Experiment studies show that unmodeled hysteresis can be effectively suppressed by the presented control scheme, despite of the low sampling frequency. Whereas it cannot be sufficiently compensated by the sole \( H_\infty \) robust controller. The results also exhibit
that the micropositioning system with the integrated controller achieves both a better tracking error and a better tracking performance index than majority of existing works.

Because the proposed index and controller are easy to implement, they can be widely used for micro-/nanopositioning systems. In the future works, more sufficient index will be discovered and more sophisticated controller will be constructed to characterize and enhance the tracking ability of the systems, respectively.

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