Design and testing of a novel multi-stroke micropositioning system with variable resolutions

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Multi-stroke stages are demanded in micro-/nanopositioning applications which require smaller and larger motion strokes with fine and coarse resolutions, respectively. This paper presents the conceptual design of a novel multi-stroke, multi-resolution micropositioning stage driven by a single actuator for each working axis. It eliminates the issue of the interference among different drives, which resides in conventional multi-actuation stages. The stage is devised based on a fully compliant variable stiffness mechanism, which exhibits unequal stiffnesses in different strokes. Resistive strain sensors are employed to offer variable position resolutions in the different strokes. To quantify the design of the motion strokes and coarse/fine resolution ratio, analytical models are established. These models are verified through finite-element analysis simulations. A proof-of-concept prototype XY stage is designed, fabricated, and tested to demonstrate the feasibility of the presented ideas. Experimental results of static and dynamic testing validate the effectiveness of the proposed design. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4866475]

I. INTRODUCTION

Micro-/nanopositioning systems have been extensively applied in the field of ultrahigh-precision manipulation and assembly. Typical applications involve scanning probe microscopy,1 biological cell manipulation,2 and precision alignment.3 Considering that flexure-based compliant mechanisms offer several merits including no backlash, no friction, and vacuum compatibility, they have been widely employed to construct ultrahigh-precision positioning systems.4–6 Generally, a micro-/nanopositioning stage exhibits two fundamental performances in terms of resolution and stroke. Due to the limitation on sensing technique, it is challenging to achieve a fine resolution and a large stroke simultaneously.7 In practice, many applications demand a micropositioning stage with multiple strokes and resolutions at the same time. For example, in robotic microassembly tasks, micropositioning systems are desired to possess a small stroke with fine positioning resolution as well as a large stroke with coarse resolution. The former is useful for precise alignment of the end-effector with respect to the component to be assembled, the latter is needed to transport the component to the assembly destination.

To cater for the requirement, a multi-stroke and multi-resolution positioning platform offers a promising solution. To generate a micropositioning stage with multiple strokes and resolutions, the multi-actuation approach is commonly employed to construct a multi-servo stage using different types of actuators. This kind of platform is generally composed of coarse substage(s) and fine substage(s) which are connected in nested or stacked manner. While the former provides a large motion stroke with a coarse positioning resolution, the latter delivers a fine positioning resolution in a smaller stroke. Dual-servo stages based on different actuation principles have been reported in the literature and on the market.1,8–10 For example, the stages with coarse and fine substages driven by voice coil motors (VCMs) and piezoelectric actuators, respectively, have been implemented for micropositioning applications.1,10

However, the major issue of a multi-servo positioning stage lies in the interference that is caused by the interaction among the substages driven by different actuators. It has been shown that the interaction behavior of a dual-servo stage can lead to an unstable open-loop control system.11 To mitigate this adverse interference effect, the mechanism and control design approaches have been developed. For instance, it has been shown that the interference behavior can be minimized by resorting to a sophisticated mechanism design.10 In addition, the interference can be alleviated by implementing a multiple-input and multiple-output control based on an interference investigation.12 Even so, the employment of different types of actuators complicates the mechanism design as well as control design processes. To overcome these issues, an alternative solution is proposed in this paper by developing a new type of multi-stroke, multi-resolution micropositioning stage using a single actuator for each working axis.

Conventionally, one actuator only delivers a single stroke along with a specific resolution. Hence, it is challenging to devise a single-drive stage with multiple strokes as well as multiple resolutions. In this research, the concept of variable stiffness mechanism is employed to accomplish the objective. In the literature, the variable stiffness concept has been exploited to develop devices with desired stiffness profiles dedicated to robot-environment interaction applications.13–15 In addition, the idea of variable stiffness has been explored to design constant-force mechanisms,16 bistable/tristable compliant mechanisms,17 and statically balanced mechanisms18 for macro- and micro-scale applications.

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Recently, a compliant micromechanism with dual-stage motion has been reported\textsuperscript{19}, which achieves two sequential displacement behaviors through the unequal stiffnesses of two sub-mechanisms. However, the concept of multiple resolutions has not been considered yet.

In this paper, a novel multi-stroke, multi-resolution micropositioning stage is designed using a single drive for each axis. The basic principle of the stage design is to devise a fully compliant variable stiffness mechanism by employing stiff and compliant flexure bearings connected in serial. The deformations of these flexure bearings are sequentially stopped by stroke limiters and unequal stiffnesses in different strokes are generated. The large and small deflections of the flexures are detected by using resistive position sensors to offer fine and coarse resolutions for different strokes. Unlike multi-servo stages, the presented technique enables the achievement of a multi-stroke motion by adopting a single actuator. This allows the elimination of the conventional interference issue. Moreover, the single-drive design permits the reduction on hardware cost and control design workload. In this paper, a single-axis and a two-axis micropositioning stages are presented for illustrations. Furthermore, both simulation and experimental investigations have been carried out to verify the proof-of-concept design of an XY multi-stroke micropositioning system.

To the knowledge of the author, the presented multi-axis stage is the first of its kind which is able to deliver multiple strokes and resolutions using a single actuator. The remainder of the paper is organized as follows. The conceptual design of the multi-stroke, multi-resolution stage is presented in Sec. II. Then, the mechanism designs of a flexure-based single-axis and two-axis compliant stages are outlined in Sec. III. Section IV details the parametric design to achieve single-axis and two-axis compliant stages are outlined in Sec. II. Then, the mechanism designs of a flexure-based single-axis and two-axis compliant stages are presented for illustrations. Furthermore, both simulation and experimental investigations have been carried out to verify the proof-of-concept design of an XY multi-stroke micropositioning system.

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The conceptual design of a micropositioning stage with multiple strokes and multiple resolutions is presented in this section.

A. Design of flexure stage with multiple strokes

The schematic of a single-axis, multi-stroke micropositioning stage is depicted in Fig. 1(a). The \( n + 1 \) masses (\( M_1 \) to \( M_{n+1} \)) are serially connected together through \( n \) flexure bearings. The \( i \)th bearing exhibits an equivalent stiffness \( K_i \) (\( i = 1 \) to \( n \)). The mass \( M_1 \) is driven by a linear actuator, and the last mass \( M_{n+1} \) is fixed at the base. To yield multiple strokes for the output platform \( M_n \), \( n \) modules of mover and stroke limiter are connected to the masses as shown in Fig. 1(a). For the \( i \)th module, the mover is linked to the mass \( M_i \), and the corresponding stroke limiter is attached to next mass \( M_{i+1} \). A specific clearance \( \delta_i \) between the \( i \)th mover and each side of the \( i \)th limiter permits a restricted bidirectional translation of the mover as well as mass \( M_i \) with respect to \( M_{i+1} \).

Assume that the relationship of \( K_1 < K_2 < \ldots < K_n \) holds. Referring to Fig. 1(a), if the actuator drives \( M_1 \) to move forward, then all of the bearings experience a common compressive force.

Initially, no mechanical mover makes contact with the corresponding stroke limiter, and the driving force \( F_{s1} \) is experienced by all of the flexure bearings. Considering the serial connections of the equivalent linear springs, the overall stiffness \( K_{s1} \) of the system in the stroke of \([0, X_1]\) can be derived

\[
K_{s1} = \left( \sum_{j=1}^{n} \frac{1}{K_j} \right)^{-1}.
\] (1)

Once a particular driving displacement \( D_1 \) is undergone, the 1st mechanical mover translates over a distance \( \delta_1 \) relative to \( M_2 \), and it contacts the forward stroke limiter. The corresponding displacement \( X_1 \) of the output platform \( M_n \) can be calculated in view of the following relationship:

\[
F_{s1} = K_{s1}D_1 = K_nX_1
\] (2)

which gives

\[
X_1 = \frac{K_1}{K_n}D_1.
\] (3)

Afterwards, if the driving continues in forward direction, the first flexure bearing \( (K_1) \) will not be deformed any more because it has been already restricted by the stroke limiter attached to \( M_2 \). Thus, \( M_1 \) can be considered to be linked to \( M_2 \) by \( K_1 \) through a rigid connection. As a result, the combined \( M_1, K_1, \) and \( M_2 \) move to right together. Under this situation, the overall stiffness of the mechanism becomes

\[
K_{s2} = \left( \sum_{j=2}^{n} \frac{1}{K_j} \right)^{-1}.
\] (4)

After the 1st mover comes into contact with the 1st stroke limiter, an additional driving displacement \( D_2 \) is produced to make \( M_2 \) translate a distance \( \delta_2 \) relative to \( M_1 \), i.e., the 2nd mover also contacts its stroke limiter. The produced displacement \( X_2 \) of the output platform \( M_n \) can be calculated in view
of the relationship
\[ F_{s2} = K_{s2}D_2 = K_nX_2 \] (5)
which gives
\[ X_2 = \frac{K_{s2}}{K_n}D_2. \] (6)

Similarly, when the \((i-1)\)th mover comes into contact with the corresponding stroke limiter, the overall stiffness of the system can be modeled as
\[ K_{ii} = \left( \sum_{j=i}^{n} \frac{1}{K_j} \right)^{-1}. \] (7)

The corresponding additional driving force is
\[ F_{si} = K_{si}D_i = K_nX_i \] (8)
which leads to an incremental value of output displacement
\[ X_i = \frac{K_{si}}{K_n}D_i. \] (9)

Hence, the forward motion range of \(M_n\) is divided into \(n\) sequential strokes of \([0, X_1], [X_1, X_1+X_2], \ldots, [X_1+X_2 + \ldots + X_{n-1}, X_1+X_2 + \ldots + X_n]\) as shown in Fig. 1(b).

On the other hand, if the output platform is driven to translate backward, its backward motion range is also divided into \(n\) intervals by the stroke limiters.

The overall motion range is partitioned into \(n\) sequential portions of \([0, |X_1|], [|X_1|, |X_1 + X_2|], \ldots, [|X_1 + X_2 + \ldots + X_{n-1}|, |X_1 + X_2 + \ldots + X_n|]\), respectively. In each of the \(n\) strokes, the overall stiffness of the system is unequal, i.e., the system stiffness exhibits \(n\) discrete values in the entire motion range.

It is notable that the mass \(M_n\) is selected as the output platform because the associated mechanical mover is the last one that will be restricted by its stroke limiter. In contrast, if the mover attached to \(M_1\) is the last restricted one, \(M_1\) can be chosen as the output platform.

In the following, a multi-resolution stage is devised based upon the aforementioned multi-stroke conceptual design.

B. Design of flexure stage with multiple resolutions

Recalling the foregoing analysis, it is observed that in the \(i\)th stroke \([|X_1 + X_2 + \ldots + X_{i-1}|, |X_1 + X_2 + \ldots + X_i]\), the deformations of \(i - 1\) flexure bearings have been already restricted by the stroke limiters, and the output displacement is contributed by the deformations of the remaining \(n - i + 1\) bearings.

In the 1st stroke \([0, |X_1|]\), the output displacement is contributed by all of the bearings. Let \(\Delta_i\) be the deformation that is experienced by the \(i\)th flexure bearing \((K_i)\). These deformations can be related by
\[ K_1\Delta_1 = K_2\Delta_2 = \ldots = K_n\Delta_n \] (10)
which describes the driving force of the actuator.
Assume that the bearings are designed to satisfy
\[ K_1 < K_2 < \ldots < K_n. \] (11)

Then, it can be deduced from (10) and (11) that
\[ \Delta_1 > \Delta_2 > \ldots > \Delta_n \] (12)
which indicates that the deformation of bearing \(i\) is greater than that of the adjacent bearing \(i + 1\).

It is known that the resistive type of sensors (e.g., resistive strain gauges or piezoresistive sensors) can be employed to measure the displacement of a flexure mechanism indirectly by detecting the induced resistance changes. If the same resistive sensor is adopted to measure two deformations with different magnitudes, the larger the deformation is, the larger the output signal will be. That is, a larger deformation results in a higher signal-to-noise ratio, i.e., higher measurement resolution. By mounting \(n\) resistive sensors on the \(n\) flexure bearings, the displacement of the output platform can be measured by these sensors. Equation (12) indicates a relationship among the resolutions provided by these sensors
\[ \text{Resol}_1 > \text{Resol}_2 > \ldots > \text{Resol}_n. \] (13)

Concerning the 1st stroke, the 1st resistive sensor which is attached on the 1st flexure bearing provides the best resolution for the output displacement measurement. While in the 2nd stroke, the deformation of the 1st bearing remains unchanged, and hence, the 1st sensor is saturated since its reading does not change. So, the 2nd sensor mounted on the 2nd bearing provides the best resolution. For the last stroke interval \(n\), the \(n\)th sensor delivers a worst resolution while the other sensors are all saturated. In most practical applications, a fine resolution is needed in a small stroke, and a worse resolution is acceptable for a larger stroke. Hence, it makes sense to design the multiple strokes as
\[ X_1 < X_2 < \ldots < X_n. \] (14)

By this way, a micropositioning stage with multiple strokes as well as multiple resolutions is devised. Specifically, the higher and lower resolutions are generated in the smaller and larger motion strokes, respectively.

III. FLEXURE-BASED COMPLIANT MECHANISM DESIGN

In this section, the mechanism design of flexure-based micropositioning stages with dual strokes and resolutions is presented as illustrations. Note that the design can be easily extended to obtain more than two strokes/resolutions.

In the literature, various types of flexible elements are available for a compliant mechanism design. To devise a micropositioning stage with long stroke, the leaf flexure is employed in this research. In particular, two basic flexure elements as shown in Figs. 2(a) and 2(b) are adopted to design the bearings. Element #1 consists of two fixed-guided leaf flexures which experience the identical deformation. Element #2 is composed of two modules of compound parallelogram flexure (CPF). The element #2 involves eight fixed-guided leaf flexures with the same dimensions. Applying a force \(F\) at the output end, both elements permit large output displacements \((\Delta_1\) and \(\Delta_2)\). Based on these two elements, the mechanism design of single- and two-axis micropositioning stages is outlined below.
A. Design of a single-axis stage

For illustration, an embodiment of a dual-stroke, dual-resolution single-axis stage is devised as shown in Fig. 3(a). The flexure bearings #1 and #2 are composed of two pieces of basic elements #1 and #2, respectively. The mover #1 is linked to the driving end of actuator, and the stroke limiter #2 is fixed at the base through two fixing holes. It is noticeable that the mover #1 and limiter #2 are mounted at the bottom of the stage. The bidirectional translations of the movers #1 and #2 are constrained by the stoke limiters #1 and #2, respectively. Both bearings #1 and #2 function as a guiding mechanism. Moreover, the bearing #2 exhibits a large transverse stiffness. Hence, in addition to the guiding role, it also tolerates a large in-plane load which is applied orthogonally to its working direction.

Additionally, the bearing #1 can be placed on either side of bearing #2 to obtain a symmetric structure as shown in Fig. 3(b). However, the two long connecting bars associated with bearing #1 add extra mass of the moving components. Thus, the design in Fig. 3(a) is more preferable to generate a higher resonant frequency.

B. Design of a two-axis stage

Moreover, a two-axis XY micropositioning stage with dual strokes and resolutions is devised as shown in Fig. 4. Owing to a symmetric design, the translations in the x and y directions follow the same principle.

The x-axis motion is produced by the bearings x1 and x2, which are composed of two elements #1 and six elements #2, respectively. The mover x1 is connected to the driving end of actuator #1, and the stoke limiter x2 is fixed at the base. In addition, the limiter x1 and mover x2 are formed by holes in the output platform. The bidirectional translations of the movers x1 and x2 are restricted by the limiters x1 and x2, respectively. The x-axis motion is guided by the bearings x1 and x2. Similarly, the y-axis motion is guided by the bearings y1 and y2 accordingly.

Note that the bearings x2 and y2 construct a decoupled XY stage, which guarantees that the translational motion in x (y) direction is independent of that in y (x) direction. Compared to previous work, the presented XY stage has a modified fixing scheme. Moreover, it owns the extra bearings x1 and x2 to accomplish the multi-stroke and multi-resolution design objectives. The stage parameters are designed in Sec. IV to achieve a specified performance.

IV. PARAMETRIC DESIGN

To produce a stage with desired performance in terms of motion ranges and resolutions, the flexure parameters call for a quantitative design. Taking the XY micropositioning stage as an example, the parametric design of the stage for the specified motion strokes and coarse/fine sensor resolution ratio are detailed in this section.

Due to a symmetric structure, the following design is conducted by considering the x-axis motion only. The same parametric design applies to the y-axis motion.
A. Design of motion strokes

1. Stiffness modeling of basic elements

First, the equivalent stiffnesses of the two basic elements as shown in Figs. 2(a) and 2(b) are modeled.

The leaf flexures in each of the two elements can be considered as fixed-guided beams as shown in Fig. 2(c). In view of the boundary conditions of the fixed-guided flexure, the following relationships hold:

\[
\frac{f l^3}{3EI} - \frac{m l^2}{2EI} = \mu, \tag{15}
\]

\[
\frac{f l^2}{2EI} - \frac{m l}{EI} = 0, \tag{16}
\]

where \(E\) is the Young’s modulus of the material, \(I = bh^3/12\) is the moment of inertia of the cross section about the neutral axis, \(b\) denotes the thickness of the material, and \(\mu\) represents the transverse deflection of the flexure as denoted in Fig. 2(c).

An inspection of (15) and (16) allows the generation of the relation

\[
f = \frac{2m}{l} = \frac{Ebh^3 \mu}{l^3}, \tag{17}
\]

which gives the linear stiffness of one leaf flexure

\[
K_0 = \frac{f}{\mu} = \frac{Ebh^3}{l^3}. \tag{18}
\]

In addition, taking into account that element #1 as shown in Fig. 2(a) is composed of two leaf flexures which are connected in serial, its equivalent stiffness can be derived below

\[
K_{e1} = \frac{K_0}{2} = \frac{Ebh^3}{2l_1^3}, \tag{19}
\]

where \(l_1\) and \(h_1\) describe the length and width of the two leaf flexures in element #1, respectively.

Recalling that the element #2 [see Fig. 2(b)] consists of two CPF modules, its stiffness can be derived as follow: \(\text{Sec.22}\)

\[
K_{e2} = \frac{Ebh^3}{2l_2^3}, \tag{20}
\]

where \(l_2\) and \(h_2\) represent the length and width of the eight leaf flexures belonging to element #2, respectively.

2. Stress and deflection calculation

Since the bending deformation dominates each leaf flexure, the minor axial deformation is neglected in the analytical modeling in this paper.

Considering the bending deformation of the leaf flexure alone, the maximum stress \(\sigma_{\text{max}}\), i.e., the yield stress \(\sigma_y\), occurs around the two terminals once the flexure undergoes the maximum moment \(m_{\text{max}}\). The stress is calculated as

\[
\sigma_{\text{max}} = \frac{m_{\text{max}} h}{2l} = \sigma_y, \tag{21}
\]

which gives the value of the maximum moment

\[
m_{\text{max}} = \frac{\sigma_y bh^2}{6}, \tag{22}
\]

where \(m_{\text{max}}\) is induced by the applied force \(f_{\text{max}}\).

By replacing the moment \(m\) with \(m_{\text{max}}\) in (17), the maximum deflection of the leaf flexure is generated

\[
\mu_{\text{max}} = \frac{\sigma_y l^2}{3Eh}. \tag{23}
\]

It can be deduced that the deflections of the leaf flexures in bearings \(x_1\) and \(x_2\) are equal to one-half and one-quarter of the deformations of the bearings \(x_1\) and \(x_2\), respectively. That is,

\[
\mu_1 = \frac{\Delta_1}{2}, \tag{24}
\]

\[
\mu_2 = \frac{\Delta_2}{4}. \tag{25}
\]

3. Stiffness modeling of the XY stage

Recalling that the bearings \(x_1\) and \(x_2\) are the parallel connections of two elements #1 and six elements #2, respectively, their stiffnesses can be derived in view of (19) and (20)

\[
K_1 = 2K_{e1} = \frac{Ebh^3}{l_1^3}, \tag{26}
\]

\[
K_2 = 6K_{e2} = \frac{3Ebh^3}{l_2^3}. \tag{27}
\]

In view of (1) and (4) along with \(n = 2\), the equivalent stiffnesses of the XY stage in the two strokes can be expressed as

\[
K_{s1} = \left( \frac{1}{K_1} + \frac{1}{K_2} \right)^{-1}, \tag{28}
\]

\[
K_{s2} = K_2. \tag{29}
\]

4. Smaller stroke design

Assume that the two motion ranges are \([0, |X_1|]\) and \([|X_1|, |X_1 + X_2|]\), respectively. First, the absolute deformation of the bearing \(x_1\) is derived as follows:

\[
\delta_1 = D_1 - X_1, \tag{30}
\]

where \(D_1\) is the driving displacement of the actuator.

Taking into account (3) and (28), the required driving displacement from actuator can be obtained

\[
D_1 = \left( 1 + \frac{K_2}{K_1} \right) X_1. \tag{31}
\]

Then, inserting (31) into (30), a fundamental operation gives

\[
X_1 = \frac{K_1}{K_2} \delta_1. \tag{32}
\]
In view of (26) and (27), the above (32) can be further expressed into the form

$$X_1 = \frac{h_1^3 l_1^3}{3h_2^2 l_1^3} \delta_1$$  

(33)

which indicates that the smaller motion range value $X_1$ is governed by the clearance $\delta_1$ as well as parameters $l_1$, $h_1$, $l_2$, and $h_2$ of the two flexure bearings.

Meanwhile, to avoid plastic deformation of the flexures, the stress caused by the deflection $\Delta_1$ of the bearing $x_1$ should not exceed the yield stress of the material. Considering (23) and (24), the allowable maximum deformation of bearing $x_1$ can be computed below

$$\Delta_{1,\text{allow}} = \frac{2\sigma_1 l_1^2}{3E h_1},$$

(34)

where $\sigma_1$ denotes the yield stress of the material.

In practice, $\Delta_1$ is constrained by the clearance $\delta_1$ between the mover $x_1$ and the stoke limiter $x_1$. Hence, this clearance should be designed to meet the following relation:

$$\delta_1 \leq \Delta_{1,\text{allow}}.$$  

(35)

Taking into account (33) and (35), the upper limit of the one-sided smaller stroke is determined by

$$X_{1,\text{max}} = \frac{2\sigma_1 h_1^2 l_1^3}{3E h_2^2 l_1^3}.$$  

(36)

5. Larger stroke design

Regarding the larger motion stroke $X_2$, it is produced by the deformation of bearing $x_2$ solely because the deformation of bearing $x_1$ is stopped by the limiter $x_1$. In view of (6) (with $n = 2$) and (29), the required driving displacement from the actuator is derived as

$$D_2 = X_2,$$  

(37)

Note that the bearing $x_2$ is deformed in both smaller and larger strokes, and the overall deformation of the bearing $x_2$ is constrained by the clearance $\delta_2$ between the mover $x_2$ and the stoke limiter $x_2$. That is, the overall motion range of the stage is limited by $\delta_2$. Hence, the one-sided larger stroke can be computed as

$$X_2 = \delta_2 - X_1.$$  

(38)

Considering (23) and (25), the allowable maximum deformation of the bearing $x_2$ can be calculated as

$$\Delta_{2,\text{allow}} = \frac{4\sigma_2 l_2^2}{3E h_2}.$$  

(39)

Practically, $\Delta_2$ is constrained by the clearance $\delta_2$. Hence, to avoid plastic deformation of the flexures associated with bearing $x_2$, the clearance $\delta_2$ should be designed to meet the condition

$$\delta_2 \leq \Delta_{2,\text{allow}}.$$  

(40)

In view of (38) and (40), once the smaller stroke $X_1$ is specified, the upper limit of the one-sided larger stroke is determined by

$$X_{2,\text{max}} = \frac{4\sigma_2 l_2^2}{3E h_2} - X_1.$$  

(41)

B. Design of coarse/fine sensor resolution ratio

In this research, two sets of resistive strain gauges are employed to measure the displacement of the compliant stage in the smaller and larger motion strokes, respectively. To enhance the sensor sensitivity, the strain gauges are mounted around the places of largest stress of the leaf flexures related to the bearings $x_1$ and $x_2$, respectively, as depicted in Fig. 3.

Assume that the output displacement of the XY stage in each axis is measured by the two strain gauge sensors through two Wheatstone bridge circuits as shown in Fig. 5. The bridge output voltages can be calculated approximately as follows:

$$V_o \approx \frac{kV_s}{4R} \times dR,$$  

(42)

where $k = 1, 2,$ and 4 represent the situations of quarter-, half-, and full-bridge circuits, respectively. Besides, $V_s$ is the supply voltage. $dR$ and $R$ represent the change and nominal values of the gauge resistance, respectively.

The gauge factor of a strain gauge is expressed as

$$S = \frac{dR/R}{\varepsilon},$$  

(43)

where $\varepsilon$ is the strain value that is induced by the deformation of the flexure bearing. The strain $\varepsilon$ is related to the experienced stress $\sigma$ of the flexure by

$$\sigma = E \varepsilon,$$  

(44)

where $E$ is the Young’s modulus of the material.

Considering (23) along with $\sigma_2$ replaced by $\sigma$, a relationship between the guided deflection $\mu$ and stress $\sigma$ is obtained

$$\mu = \frac{\sigma l_2^2}{3E h_2}.$$  

(45)

In view of (42)–(45), the relationship between the circuit output voltage and the flexure deflection can be derived

$$\mu = \frac{4l_2^2 V_o}{3hSV_s}.$$  

(46)

FIG. 5. Wheatstone bridge circuit with one strain gauge and three fixed resistors. $V_s$ and $V_o$ are supply voltage and bridge output voltage, respectively.
Recalling (24) and (25), the ratio of output voltages \( V_{o1} \) and \( V_{o2} \) of the two strain gauges which are attached on the two bearings (x1 and x2) is obtained from (46)

\[
\eta = \frac{V_{o1}}{V_{o2}} = \frac{2h_2 l^2_2 \Delta_1}{h_2 l^2_1 \Delta_2} = \frac{6h_2^2 l_1}{h_1^2 l_2}
\]

(47)

Taking into account (10) as well as (26) and (27), the relation (47) further reduces to

\[
\eta = \frac{V_{o1}}{V_{o2}} = \frac{6h_2^2 l_1}{h_1^2 l_2}
\]

(48)

which indicates that the output voltage ratio of the two strain gauges is determined by the flexure parameters \( l_1, h_1, l_2, \) and \( h_2 \) of the XY stage. By properly selecting these flexure parameters, a relation of \( \eta > 1 \) can be obtained. That is, a higher signal-to-noise ratio will be produced by the strain gauge x1. It follows that a coarse/fine resolution ratio \( \eta \) can be achieved by the two strain gauge sensors.

### C. Actuation issue consideration

In this research, a VCM is adopted to produce a sufficient driving displacement. To create an entire motion range of \( 2(X_1 + X_2) \), a driving displacement of \( D_{\text{input}} = 2(D_1 + D_2) \) is needed, which can be calculated using (31) and (37).

This input displacement should not exceed the stroke \( D_{\text{actuator}} \) of the selected actuator, i.e.,

\[
D_{\text{input}} = 2 \left( 1 + \frac{3h_2 l_1}{h_1 l_2} \right) X_1 + 2X_2 \leq D_{\text{actuator}}
\]

(49)

which is calculated in consideration of (26) and (27).

In addition, the stage should be designed to be compliant enough so that the elastic energy can be overcome by the VCM. Taking into account the assumption (11), it is deduced from (28) and (29) that \( K_{s1} < K_{s2} \). That is, the stiffness in the smaller stroke is lower than that in the larger one. Hence, the maximum force is needed to produce the extremum of the overall motion range. The required maximum driving force can be calculated as follows:

\[
F_{\text{max}} = K_{s2}(X_1 + X_2) \leq F_{\text{actuator}},
\]

(50)

where \( F_{\text{actuator}} \) denotes the maximum driving force of the VCM actuator.

Substituting (29) and (27) into (50), it can be derived that

\[
F_{\text{max}} = \frac{3Ebh_2^3}{l_2^3}(X_1 + X_2) \leq F_{\text{actuator}}.
\]

(51)

Hence, (49) and (51) provide the guidelines for the stage parameter design in consideration of the stroke and force limits of the actuator.

### V. STAGE PERFORMANCE ASSESSMENT

The foregoing parametric design procedures offer the guidelines (35), (36), (40), (41), (48), (49), and (51) for the stage parameter design. As a case study, an XY micropositioning stage is designed to produce the smaller and larger motion strokes of \( X_1 = Y_1 = 0.2 \) mm and \( X_2 = Y_2 = 2.3 \) mm, respectively, as well as a coarse/fine resolution ratio of \( \eta = 5.6 \) in each working axis. The selected VCM provides a stroke of 10.2 mm and maximum driving force of 29.2 N. The stage parameters are designed as shown in Table I.

### A. Analytical model evaluation results

The analytical models predict that the stage parameters allow the generation of \( X_{1\text{max}} = 2.10 \) mm, \( X_{2\text{max}} = 11.79 \) mm, \( D_{\text{input}} = 8.02 \) mm, and \( F_{\text{max}} = 25.0 \) N, which all satisfy the aforementioned design criteria. The overall motion range in each axis is constrained by \( \delta_2 \) as \( \pm 2.5 \) mm.

In addition, the parametric design leads to an output voltage ratio \( \eta = \frac{V_{o1}}{V_{o2}} = 5.6 \). Therefore, the relationship of signal-to-noise ratios (SNRs) of the two strain gauges can be predicted as \( \text{SNR}_{1} = \text{SNR}_{2} = \eta = 5.6 \). It means that the coarse/fine resolution ratio of the two strain gauge sensors is \( \text{Resol}_{1} = \frac{\delta_2}{\delta_1} = \eta = 5.6 \).

That is, the resolution in the smaller stroke has been improved by 5.6 times as compared with that in the larger motion stroke of each working axis.

### B. FEA simulation results

To verify the accuracy of the established analytical models, FEA simulations are carried out using ANSYS software package.

#### 1. Static analysis results

First, the static performance of the XY stage is evaluated with static structural FEA. The simulations are carried out by applying an input force at the input end to produce the smaller and larger motions, respectively. Owing to a symmetric structure of the stage, only the x-axis results are presented below.

With an input force of 10 N applied at the input end, the performance in the smaller stroke is tested. The deformation and stress distribution results of the FEA simulation are illustrated in Figs. 6(a) and 6(b), respectively. To generate a motion stroke of \( \pm 0.2 \) mm, the required driving displacement is \( \pm 1.80 \) mm. Considering the FEA result as the benchmark, it is seen that the analytical model result of the input displacement (\( \pm 1.71 \) mm) is 5% lower than that of FEA. Moreover, simulation results reveal that the maximum value of the smaller range is \( X_{1\text{max}} = 2.02 \) mm. The discrepancy between the

### TABLE I. Main parameters of a dual-stroke and dual-resolution XY micropositioning stage.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 )</td>
<td>0.5</td>
<td>mm</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>0.35</td>
<td>mm</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>21</td>
<td>mm</td>
</tr>
<tr>
<td>( b )</td>
<td>10</td>
<td>mm</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>1.5</td>
<td>mm</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>2.5</td>
<td>mm</td>
</tr>
<tr>
<td>( E )</td>
<td>71.7</td>
<td>GPa</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>503</td>
<td>MPa</td>
</tr>
</tbody>
</table>

FIG. 6. Static FEA simulation results with an input force applied. (a) Deformation results; (b) stress distribution results.

analytical model and FEA simulation is less than 0.5%. In addition, Fig. 6(b) confirms that the maximum stresses occur around the ends of the leaf flexures associated to the bearings \( x_1 \) and \( x_2 \), respectively.

Regarding the larger motion stroke, FEA results indicate that the maximum deflection \( X_{\text{max}} \) is 12.10 mm. As compared with the FEA prediction, the analytical model underestimates the larger stroke by 2.6%. In addition, to generate a one-sided overall motion range of 2.5 mm, the FEA predicts that the required maximum driving force is 27.5 N. Hence, the difference between the analytical and FEA results for the driving force assessment is 9%.

The aforementioned discrepancies between the analytical model and FEA simulation results are all lower than 10%. These discrepancies mainly arise from the adopted assumption for the analytical models, which only consider the bending deformations of the leaf flexures. The model accuracy can be enhanced by taking into account both bending and axial deformations of the flexures.

2. Dynamic analysis results

The modal analysis is performed to evaluate the dynamic performance of the XY stage. In particular, the frequencies and shapes of the first-five resonant modes are shown in Fig. 7.

It is observed that the first-two mode shapes indicate the translations along the \( x \)- and \( y \)-axes with similar resonant frequencies of 35.2 and 35.3 Hz, respectively. The third mode is attributed to the in-plane rotation of the stage output platform. It is found that the third resonant frequency (72.5 Hz) is more than twice higher than the first-two fundamental frequencies. The forth and fifth resonant modes are induced by the translations of the stage along the two working axes with higher frequencies of 94.3 and 94.4 Hz, respectively. Moreover, the simulation results reveal that the remaining resonant modes are mainly induced by the in-plane and out-of-plane deflections of the internal leaf flexures related to the two bearings.

VI. PROTOTYPE DEVELOPMENT AND EXPERIMENTAL STUDIES

In this section, a prototype XY micropositioning stage is described and experimental testing results are presented.

A. Prototype development

Fig. 8 shows a photo of the developed prototype stage. The stage is fabricated from a plate of Al 7075 alloy by the wire-electrical discharge machining process. The stage possesses a dimension of 240 mm \( \times \) 240 mm \( \times \) 10 mm. Concerning the actuator, two VCMs (model: NCC04-10-005-1A, from H2W Techniques, Inc.) are chosen to provide sufficient large stroke of 10.2 mm and output force of 29.2 N. Each VCM is driven by the NI-9263 analog output module (from National Instruments Corp.) through a VCM driver. The stage output displacements in the small and large strokes in each working axis are measured by two resistive strain gauges (model: SGD-3/350-LY13, from Omega Engineering Ltd.). This type of strain gauge possesses a nominal resistance
FIG. 8. Fabricated prototype of the XY micropositioning stage. The laser sensors are used for the purpose of calibration only.

of 350 $\Omega$, a gauge factor of 2, and a dimension of 7 mm $\times$ 4 mm.

To measure the quarter Wheatstone bridge (see Fig. 5) output, the NI-9945 quarter-bridge completion accessory is used to complete the 350 $\Omega$ sensor. The NI-9945 contains three high-precision resistors of 350 $\Omega$. Generally, the output voltage of a Wheatstone bridge is very small. In order to measure the bridge output accurately, a voltage amplifier can be adopted to pre-amplify the sensor output. However, the sensor noise will be amplified at the same time. Alternatively, the bridge output can be acquired by using a high-resolution data acquisition device directly. In this research, the NI-9237 bridge input module is employed, which provides a resolution of 24-bit. It is able to acquire the quarter-bridge output signal directly and produce a maximum voltage output of $\pm 25$ mV per volt of excitation voltage. For example, with an excitation voltage of 3.3 V, the maximum bridge output is $\pm 82.5$ mV.

For the calibration of the strain sensors, two laser displacement sensors (model: LK-H055, from Keyence Corp.) with a resolution of 25 nm and measurement range of 20 mm are employed. In addition, a NI cRIO-9022 real-time controller combined with NI-9118 chassis is adopted as the controller hardware. NI LabVIEW software is employed to implement a deterministic real-time control of the micropositioning system. A sampling rate of 5 kHz is adopted in this research.

B. Static performance testing

The static performances of the XY stage in both $x$- and $y$-axes have been tested. Since the results of the two axes are very similar, only the $x$-axis results are detailed below.

First, the two sensor outputs are calibrated. By applying a sinusoidal voltage signal with frequency of 0.1 Hz and amplitude of 0.5 V to the driver of VCM #1, the stage output position in the $x$-axis is measured by the laser sensor as shown in Fig. 9(a). The output voltages of the two strain gauges are depicted in Fig. 9(b). It is observed that the strain gauges $x_{1}$ and $x_{2}$ produce the voltage ranges of $5.2087 \times 10^{-4}$ and $1.0702 \times 10^{-5}$ V, respectively. The two strain gauge sensors are calibrated by comparing their output voltages to the laser sensor output. The calibrated sensors provide the output displacements as shown in Figs. 9(c) and 9(d), respectively. It is obvious that the output of the sensor $x_{2}$ is noisier than that of the sensor $x_{1}$.

The resolutions of the two strain sensors are limited by their noises. In this work, the noise mainly comes from the electric noise of the digital to analog converter. The motion of the micropositioning stage lower than the sensor noise level cannot be detected by the sensor. With zero voltage input, the noises of the two strain sensors have been recorded as shown Figs. 10(a) and 10(c), respectively. The histograms of the two sensor noises are depicted in Figs. 10(b) and 10(d), respectively. It is found that the noises follow normal distributions closely with the standard deviations ($\sigma$) of 1.625 and 8.517 $\mu$m, respectively. To quantify the noise level, the $\sigma$ value is adopted as the sensor resolution.24, 25 Then, the resolution ratio of the coarse and fine sensors is derived as 5.24, which is 6.4% lower than the analytical prediction of 5.6. This discrepancy mainly arises from the fabrication errors of the stage parameters and the mounting errors of the strain gauges. The experimental results reveal that the resolution in
the smaller stroke has been improved by 5.24 times compared to that in the larger motion stroke.

Next, the magnitudes of the smaller and larger motion ranges are tested by applying a sinusoidal signal with an amplitude of 7.5 V as shown in Fig. 11(a). The output displacements as measured by the two strain sensors are shown in Figs. 11(b) and 11(c), respectively. It is found that the strain sensor \( x_1 \) saturates at the boundaries of the interval \([-230 \mu m, 225 \mu m]\), which represents the smaller motion stroke of the stage. In the larger stroke of \([-2452 \mu m, -230 \mu m]\) and \([225 \mu m, 2554 \mu m]\), the output displacement is measured by the strain sensor \( x_2 \) alone, which provides a worse resolution than sensor \( x_1 \) as tested earlier. The overall motion range \([-2452 \mu m, 2554 \mu m]\) is constrained by the actual clearance parameter \( \delta_2 \) between the mover \( x_2 \) and the stoke limiter \( x_2 \).

C. Dynamic performance testing

The dynamic performances of the XY stage are examined by the frequency response method. To test the \( x \)-axis performance, a swept-sine signal with the amplitude of 0.03 V and frequency range of 1–500 Hz is applied to drive VCM #1. The frequency responses of the stage output position in the \( x \)-axis are shown in the Bode plots in Fig. 12, which are generated from the outputs of the laser sensor and two strain sensors.

Owing to a small motion amplitude, all of the three sensors are able to capture the first-two modes at 17.5 and 29.0 Hz, respectively. As compared with the FEA simulation results (35.2 and 94.3 Hz) of the first-two modes in the \( x \)-axis working direction, the experimental results are much lower. The discrepancy between the simulation and experimental results for the resonant frequencies mainly comes from the added mass of the connected mover and the moving coil of VCM, which is not considered in FEA simulation. The resonant frequency can be enhanced by reducing the mass of the moving components.

While both the laser sensor and the strain sensor \( x_2 \) predict the resonance at 17.5 Hz (mode 1) as the dominant resonant mode, the strain sensor \( x_1 \) detects the 29.0-Hz (mode 2) resonance as the dominant one. The reason can be explained by inspecting the two mode shapes as shown in Figs. 7(a) and 7(d), respectively. It is observed that the strain induced in mode 2 is much larger than that in mode 1 at the mounting place of sensor \( x_1 \). As a result, the strain sensor \( x_1 \) predicts the resonant mode 2 as the dominant mode alternatively. In contrast, the correct dominant resonant mode can be assessed by the strain sensor \( x_2 \) as well as the laser sensor.
D. Discussion

For a better understanding of the results, a comparison of the desired and actual main performances of the XY stage is shown in Table II, where the results of both x- and y-axes are presented. It is found that the smaller strokes in both axes are larger than the design objective of ±0.2 mm. The overall motion ranges in both axes are slightly larger than the design specification of 5.0 mm, which also indicates that the driving displacement lies within the stroke of the employed VCM actuators. In addition, the phenomenon that both the smaller and larger bidirectional motion strokes in each axis are not exactly symmetric with respect to zero is attributed to the manufacturing errors and the unequal clearances between the movers and each side of stroke limiters. The discrepancy between the desired and actual resolution ratios for the two strokes in each axis comes from the manufacturing errors as well as the variation on the attached adhesive layers of the two strain gauges.

Fig. 12 exhibits that the strain sensors x1 and x2 are able to detect some minor resonant modes at frequencies higher than 40 Hz. However, these modes cannot be detected by the laser sensor. The reason why the laser sensor cannot capture these resonant modes can be explained by examining the measurement principles of the sensors. Specifically, the two strain sensors measure the displacement of the output platform indirectly by monitoring the strain deformation of the internal flexures associated to the two compliant bearings. Whereas the laser sensor is fixed at the base and directly measures the output displacement of the platform through a sensor target, which is attached on the platform. As predicted by the FEA simulation, the first-two resonant modes in each working axis are attributed to the translations of the output platform. Thus, they are detected by all of the three sensors. However, the minor resonant modes (higher than 40 Hz) are not related to the output platform, but contributed by the deformations of the intermediate flexures alone. Hence, these modes cannot be detected by the laser sensor which monitors the output platform displacement only. The experimental results verified the resonant mode shapes evaluated by the FEA simulation.

It is notable that the static and dynamic performances of the stage have been characterized under open-loop drive status. This motivates a further work on closed-loop control to realize a precision control of the developed compliant stage. Hence, there is a room for further improvement on the stage performance. Even so, the conducted experiments validated the effectiveness of the proposed conceptual design of a multi-stroke, multi-resolution micropositioning stage with a single drive in each axis. In the reported case study, the larger stroke is 10 times larger than the smaller stroke, and the positioning resolution in the smaller stroke is about 5 times higher than that in the larger stroke.

The presented micropositioning system offers fine resolution only in the smaller stroke around the center of the workspace. At the limit of the smaller stroke, the impact between the mover and the limiter may cause vibration of the flexures. The impact can be alleviated by applying low-frequency input signal as depicted in the experimental results, as shown Fig. 11. An input signal of higher frequency will induce clear impact vibration. Even so, unlike the interference in a dual-servo stage, where the interaction exhibits during the entire stroke of the fine stage, the impact effect only exists at the two limits of the smaller stroke in the presented stage. To eliminate the impact, a trajectory with low positioning speed at the two limits can be planned.

It is noted that the proposed design poses certain limitations in terms of bandwidth for the coarse and fine positioning. In the conventional multi-servo stage, the fine positioner exhibits a much higher bandwidth than the coarse positioner. In the reported stage, the fine motion stroke is generated by carrying the full mass of the stage, which severely limits the bandwidth. Hence, a low-speed motion is expected for the smaller stroke. In addition, the condition as shown in (14) indicates that the fine and coarse positioning are achieved in the first and last strokes, respectively. This seems contradict the requirement of a typical multi-stroke application, where the coarse positioning is realized in the first stroke and the fine positioning is obtained in the last stroke. The proposed multi-stroke stage can be employed in potential applications, which require a fine and slow-speed alignment around the home position and a coarse and fast-speed positioning in the larger stroke. For example, in microgripper-based assembly tasks, the proposed stage can be used to precisely align the microgripper to grasp an object initially, and then quickly transport the object to the destination.

Moreover, it is observed that the multi-resolution positioning is implemented using resistive position sensors. Actually, there are better position sensors which can provide better resolution at fine strokes. For example, the used laser displacement sensors deliver a better resolution than the strain gauges. However, the sensor head and controller are bulky, which precludes their use in compact system. The same issue exists in other fine sensors such as laser interferometer. In addition, the capacitance-based sensors are more compact, which measure the displacement between two parallel plates. In this research, the resistive strain gauges have been employed since they enable an embedded and compact design. Experimental results show that they are able to provide different resolutions for different strokes. It is notable that the resolutions of the strain sensors are tested using the quarter-bridge circuits in this research. The sensor resolutions can be improved by twofold or fourfold by adopting half- or full-bridge circuits, respectively. Moreover, the resolution of the strain sensors can be further enhanced by removing high-frequency noises using low-pass filters. Other types of sensors such as piezoresistive sensors can also be exploited to improve the

<table>
<thead>
<tr>
<th>Performance</th>
<th>Desired x- and y-axes</th>
<th>Actual x-axis</th>
<th>Actual y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smaller stroke (mm)</td>
<td>[-0.2, 0.2]</td>
<td>[-0.225, 0.230]</td>
<td>[-0.210, 0.223]</td>
</tr>
<tr>
<td>Larger stroke (mm)</td>
<td>[-2.5, -0.2]</td>
<td>[-2.452, -0.225]</td>
<td>[-2.560, -0.210]</td>
</tr>
<tr>
<td>Coarse/fine resolution ratio</td>
<td>5.0</td>
<td>5.24</td>
<td>5.32</td>
</tr>
</tbody>
</table>
positioning resolutions. The ideas that are presented in this paper can be extended to the design of other types of micropositioning stages (e.g., rotary stages) as well.

VII. CONCLUSIONS

The conceptual design and verification of a single-drive compliant micropositioning system with multiple strokes and resolutions in each axis has been reported in this paper. Based on the concept of variable stiffness mechanism, a proof-of-concept design of a novel flexure-based XY stage is presented. Analytical models have been established to predict the motion strokes, coarse/fine resolution ratio as well as driving force and stroke requirements, which have been verified by FEA simulations and experimental studies. Experimental results show that the developed stage is able to produce multiple strokes using a single linear motor, and the same kind of strain gauge sensor is capable of providing fine and coarse resolutions in the small and large strokes, respectively. The design theory can be extended to include parallel connection of the flexure bearings. In the future, suitable control schemes will be devised to accomplish a precise positioning for related applications.

ACKNOWLEDGMENTS

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