Enhanced discrete-time sliding mode strategy with application to piezoelectric actuator control

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Abstract: This study presents a new discrete-time sliding mode control (DSMC) scheme with applications to precise motion control of piezoelectric actuators. Different from existing DSMC algorithms whose implementations rely on the construction of state observers for providing the state feedback, a simple yet effective DSMC strategy is developed based on a discrete-time model without using the state observer. Hence, one distinctive feature of the proposed DSMC lies in that it is very easy to implement. Only a second-order plant model is needed whereas the modelling of piezoelectric non-linearities is not required, which further simplifies the practical implementation process. The local stability of the closed-loop system is proved in theory and the effectiveness of the DSMC is demonstrated by several experimental studies. Results show that the DSMC strategy is superior to proportional-integral-derivative control in terms of transient response speed, positioning accuracy and robustness against external disturbances. The reported method can be extended for precise motion control of other second-order systems as well.

1 Introduction

Micro/nanopositioning is a crucial technique for micro/nanomanipulation and assembly systems such as scanning probe microscopes [1]. As a typical smart material-based actuator, piezoelectric actuator exhibits the merits of sub-nanometer positioning resolution and rapid response speed. Hence, piezoelectric actuators have been extensively adopted in the aforementioned applications. For instance, piezoelectric bimorph actuators are usually employed to construct a microgripper dedicated to micro/nanomanipulation tasks [2, 3]. The major challenge of using piezoelectric actuator for micro/nanopositioning lies in how to overcome the piezoelectric non-linearities in terms of hysteresis and creep effects.

In the last two decades, numerous voltage-driven strategies have been developed to suppress the piezoelectric non-linearities. Generally, these control schemes fall into two categories in terms of hysteresis model-based and hysteresis model-free approaches. Although the hysteresis effect can be well compensated for using the first method [4], that is, modelling the hysteresis behaviour and constructing an inverse hysteresis model-based feedforward control, the result is vulnerable to the hysteresis model error [5]. Moreover, the creep phenomenon needs an extra treatment in such scheme. Thus, the second approach, which is dependent on the design of a feedback control [6–8], is more attractive from the implementation point of view. Particularly, sliding mode control (SMC) has demonstrated its potential in precise motion control in the presence of external disturbances and model uncertainties. Hence, SMC has drawn extensive attentions of recent investigations [9–12].

In order to implement SMC on a sampled-data system, the discrete-time SMC (DSMC) is more attractive [13–15]. Generally, the DSMC can be categorised into state-based and output-based methods. The former is developed based on the system state or state error [16–18], whereas the latter is realised on the base of the system output or output error [19, 20]. Usually, the implementation of both methods requires the state feedback of the system. However, in majority of practical situations, only the position information of a piezoelectric actuator system is provided by the displacement sensor. Hence, a state observer is indispensable for the practical realisation of DSMC [16, 18–20], which complicates the control design procedure. Furthermore, an improperly designed state observer may cause instability of the system. In this sense, it is desirable to eliminate the use of state observer. However, only limited works have been made towards this issue. In the literature, an input–output-based adaptive DSMC was proposed in the previous work [21], which is based solely on input and output data. However, the controller was developed for a first-order model with dead time. It is unsuitable for a piezoelectric actuator system which typically possesses a higher-order plant model preceded by complicated non-linearity.

To this end, the motivation of this research is to develop a simple DSMC scheme without using the complicated hysteresis model and state observer for precise motion control of a piezoelectric actuator. Specifically, the unmodelled non-linearity effects are treated as a lumped perturbation and the perturbation is estimated by resorting to a one-step delayed estimation technique. Furthermore, the avoidance of state observer is realised by developing a new DSMC...
based on a discrete-time second-order dynamics model of the system. The local stability of the closed-loop system is proved theoretically and the effectiveness of the proposed scheme is validated through experimental investigations.

To the knowledge of the author, the proposed scheme is the most straightforward approach on the basis of DSMC framework dedicated to piezoelectric actuator control.

The remainder of this paper is organised as follows. Based on the dynamics model of a piezoelectric actuator, the motion control problem is formulated in Section 2. The design procedure of a new DSMC scheme is detailed in Section 3 along with a stability analysis. Section 4 presents the experimental setup and comparative studies of the proposed strategy with respect to the popular proportional-integral-derivative (PID) control through several experimental investigations. Detailed discussions and future work are also provided therein. Section 5 concludes this paper.

2 Dynamics model and problem formulation

Piezoelectric bimorph actuators are usually employed to construct microgrippers for performing delicate manipulation tasks. In this work, a multi-layer piezoelectric bimorph actuator as shown in Fig. 1 is picked out for a detailed analysis. The cantilever suffers from an excitation voltage \( u \) and a contact force \( F \) which is applied by the manipulated object. The overall output displacement is described by \( x \). To guarantee a precise positioning, both piezoelectric non-linearities and external disturbances call for a suitable control technique.

The dynamics model of a piezoelectric actuator driven by an input voltage can be established as [18]

\[
M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = Du(t) + P(t)
\]  

(1)

where \( t \) is the time variable, parameters \( M, B, K \) and \( x \) represent the mass, damping coefficient, stiffness and output displacement of the actuator, respectively; \( D \) is the piezoelectric coefficient and \( u \) denotes the input voltage. In addition, it is assumed that the parameters \( M, B, K \) and \( D \) are known by the approach of system identification. The perturbation term \( P(t) \) describes the lumped effect of piezoelectric hysteresis, creep, external force, parameter uncertainties and other disturbances. Similar treatment can be found in [11, 22]. When the driving voltage is \( u = 0 \), \( P \) does not include the piezoelectric hysteresis and creep effects.

The existing continuous-time and DSMC are usually designed based on the system state. Thus, a state observer is required for the implementation of SMC schemes. In the present research, a simple DSMC strategy is devised to eliminate the use of state observer.

First, dividing both sides of (1) by \( M \) results in

\[
m\ddot{x}(t) + b\dot{x}(t) + kx(t) = du(t) + p(t)
\]  

(2)

where \( m = 1, b = (B/M), k = (K/M), d = (D/M) \) and \( p(t) = (P(t)/M) \).

Then, the continuous-time model (2) is discretised by adopting a small sampling time \( T \). For the purpose of discretisation, several approaches are available (e.g. zero-order hold). In this research, the Euler backward difference is employed owing to its simplicity [23–25]

\[
\begin{align*}
\dot{x}(t) &\simeq \frac{1}{T}[x(kT) - x(kT - T)] \\
\ddot{x}(t) &\simeq \frac{1}{T^2}[x(kT) - 2x(kT - T) + x(kT - 2T)]
\end{align*}
\]  

(3)

(4)

where \( k \) denotes the \( k \)th time step. Thus, the continuous-time dynamics model (2) is converted into an equivalent discrete-time form

\[
m\ddot{x}_{k-2} + \ddot{h}x_{k-1} + \ddot{k}x_k = \ddot{d}u_k + p_k
\]  

(5)

where \( x_{k-2} = x(kT - 2T) \) and

\[
\begin{align*}
\ddot{m} &= \frac{1}{T^2}, \quad \ddot{b} = -\frac{b}{T} - \frac{2}{T^2}, \quad \ddot{k} = k + \frac{b}{T} + \frac{1}{T^2}, \\
\ddot{d} &= d
\end{align*}
\]  

(6)

It is notable that the backward differences (3) and (4) are employed to discretise the continuous-time model (2), and the discretisation noises are not considered here. The discrete-time model (5) includes the delayed versions \( x_{k-2} \) and \( x_{k-1} \) of the plant output \( x_k \). Similar to other approaches such as zero-order hold, the main problem of the discretisation is the generated time delay (\( \geq T/2 \)). The time delay causes slow response in transient behaviour of the closed-loop control system [26]. In this research, the time delay is neglected since a small sampling time \( T \) will be selected.

Based on the perturbation estimation technique [27], the perturbation term \( p_k \) can be generated by its one-step delayed estimation

\[
\hat{p}_k = p_{k-1} = -\ddot{d}u_{k-1} + \ddot{m}x_{k-3} + \ddot{b}x_{k-2} + \ddot{k}x_{k-1}
\]  

(7)

Hence, the dynamics model (5) can be rewritten as

\[
m\ddot{x}_{k-2} + \ddot{h}x_{k-1} + \ddot{k}x_k = \ddot{d}u_k + \ddot{p}_k - \ddot{p}_k
\]  

(8)

where \( \ddot{p}_k = \ddot{p}_k - p_k \) is the perturbation estimation error, which can be further expressed as

\[
\ddot{p}_k = p_{k-1} - p_k \simeq -\dot{p}(t)T = -\frac{T}{M}\dot{P}(t)
\]  

(9)

Assumption 1: The first derivative of the lumped perturbation \( P(t) \) is bounded, that is, \( |\dot{P}(t)| \leq \delta \).

In view of (9) and Assumption 1, it can be deduced that \( \ddot{p}_k \) is also bounded, that is,

\[
|\ddot{p}_k| \leq \frac{T\delta}{M}
\]  

(10)

In order to overcome the error term \( \ddot{p}_k \) and to achieve a precise position control, a DSMC scheme is devised in the next section.
3 DSMC design

Although a number of DSMC algorithms have been presented in the literature [16, 18–21], the existing works are not directly applicable to the current problem. Hence, a new DSMC with integral action is developed in this section.

First, substituting the estimated perturbation term (7) into the dynamics model (8) allows the calculation of the position error

\[ x_k = \frac{1}{k} [d(u_k - u_{k-1}) + \bar{m}x_{k-3} + (\bar{b} - \bar{m})x_{k-2} + (\bar{k} - \bar{b})x_{k-1}] \]

Based on the position error \( \varepsilon_k = x_k - x_{d,k} \) where \( x_{d,k} \) is the desired position trajectory, a proportional-integral (PI) type of sliding function is defined as follows

\[ s_k = \lambda_p \varepsilon_k + \lambda_i \varepsilon_k \]

where \( \lambda_p \) and \( \lambda_i \) are the proportional and integral gains, respectively. In addition, the integral error is defined as

\[ \varepsilon_k = \sum_{i=1}^{k} \varepsilon_i = \varepsilon_k + \varepsilon_{k-1} \]

Regarding the reaching law, there are two different definitions as summarised in [28, 29]. In this research, the following definition is adopted [30]

\[ \Delta s_k = s_k - s_{k-1} = 0 \]

Considering that the equivalent control \( u_k^{eq} \) is the solution to (14), the following deductions hold

\[ \lambda_p \varepsilon_2 + \lambda_i \varepsilon_2 = s_{k-1} \]

\[ \Rightarrow (\lambda_p + \lambda_i) \varepsilon_2 + \lambda_i \varepsilon_{k-1} = s_{k-1} \]

\[ \Rightarrow (\lambda_p + \lambda_i) (x_k - x_{d,k}) + \lambda_i \varepsilon_{k-1} = s_{k-1} \]

Then, inserting (11) into (17) and ignoring the estimation error \( \hat{p}_k \), leads to the equivalent control

\[ u_k^{eq} = u_{k-1} + \frac{1}{d} \left[ \frac{k}{\lambda_d} s_{k-1} - \frac{k}{\lambda_d} \lambda_i \varepsilon_{k-1} + \bar{k} x_{d,k} \right] \]

\[ - \frac{1}{d} [\bar{m} x_{k-3} + (\bar{b} - \bar{m})x_{k-2} + (\bar{k} - \bar{b})x_{k-1}] \]

where \( \lambda_d = \lambda_p + \lambda_i \).

The equivalent control (18) represents the control action for the case of perfect disturbance estimation, that is, \( \hat{p}_k = 0 \). It takes effect in the sliding phase when the position trajectory is kept on the sliding surface \( (s_k = 0) \). However, if a large error \( \hat{p}_k \) occurs during the sliding phase, the standalone equivalent control cannot drive the position towards the sliding surface. Thus, the equivalent control is augmented by a switching control \( u_k^{sw} \) to give the total control action

\[ u_k = u_k^{eq} + u_k^{sw} \]

that is,

\[ u_k = u_{k-1} + \frac{1}{d} \left[ \frac{k}{\lambda_d} s_{k-1} - \frac{k}{\lambda_d} \lambda_i \varepsilon_{k-1} + \bar{k} x_{d,k} \right] \]

\[ - \frac{1}{d} [\bar{m} x_{k-3} + (\bar{b} - \bar{m})x_{k-2} + (\bar{k} - \bar{b})x_{k-1}] \]

\[ - \frac{\lambda_d}{d} \text{sgn}(s_k) \]

where \( \lambda_d \) is a positive control gain and \( \text{sgn}() \) denotes the signum function.

Theorem 1: For the system (8) with Assumption 1 and sliding function (12), if the controller (20) with gain \( \lambda_d \geq |\hat{p}_k| \) is employed, then the discrete sliding mode will occur with a quasi-sliding domain (QSD) width of QSD = \( (\lambda_d/\bar{\lambda}_d) k \bar{p}_k \) after a finite number of steps.

Proof: Substituting (20) into the expression of the sliding function (12), a necessary algebra operation gives

\[ s_k = \lambda_a (x_k - x_{d,k}) + \lambda_i \varepsilon_{k-1} \]

\[ = \lambda_a \left[ \frac{1}{k} [d(u_k - u_{k-1}) + \bar{m} x_{k-3} + (\bar{b} - \bar{m}) x_{k-2} + (\bar{k} - \bar{b}) x_{k-1}] \right] \]

\[ + (\bar{k} - \bar{b}) x_{k-1} - \hat{p}_k \] + \( \lambda_i \varepsilon_{k-1} \]

\[ = s_{k-1} - \frac{l_a}{k} \lambda_d \text{sgn}(s_{k-1}) - \hat{p}_k \]

Note that parameters \( \bar{k}, \lambda_d \) and \( \lambda_S \) are all positive. In the case of \( s_{k-1} \geq 0 \), it can be derived that

\[ s_k \leq s_{k-1} \quad \text{if} \quad \lambda_d \geq |\hat{p}_k| \]

Otherwise, if \( s_{k-1} < 0 \), then

\[ s_k \geq s_{k-1} \quad \text{if} \quad \lambda_d \geq |\hat{p}_k| \]

Thus, in view of (22) and (23), the following conclusion can be drawn

\[ |s_k| \leq |s_{k-1}| \quad \text{if} \quad \lambda_d \geq |\hat{p}_k| \]

Recalling that (10) holds, it can be deduced from (24) that \( s_k \) decreases monotonously, and the discrete sliding mode is reached after a finite number of steps. \( \square \)

According to (21), the change value of the sliding function can be expressed into the form

\[ s_k - s_{k-1} = - \frac{\lambda_d}{k} \lambda_s \text{sgn}(s_{k-1}) - \hat{p}_k \]

\[ = \frac{\lambda_d \lambda_s}{k} + \frac{\lambda_d}{k} \hat{p}_k \neq 0 \]

Hence, the QSD which represents the variation of the sliding function change can be determined as

\[ |s_k - s_{k-1}| \leq \frac{\lambda_d \lambda_s}{k} + \frac{\lambda_d}{k} \hat{p}_k = \text{QSD} \]
Remark 1: It is notable that only the local convergence is obtained in Theorem 1 because of the lack of knowledge of the bound value for \( p_k \). In addition, the relationship (24) represents a sufficient condition for the existence of discrete sliding mode [31]. The selection of parameter \( \lambda_S \) has no direct relation to the initial conditions. Generally, \( \lambda_S \) as well as \( \lambda_P \) and \( \lambda_3 \) can be assigned in consideration of the QSD width as represented by (26).

Remark 2: It has been shown that the relationship of \(|s_k| \leq |s_{k-1}|\) can be decomposed into two inequalities [31]

\[
\begin{align*}
(s_k - s_{k-1}) \operatorname{sgn}(s_{k-1}) &\leq 0 \\
(s_k + s_{k-1}) \operatorname{sgn}(s_{k-1}) &> 0
\end{align*}
\]

which are called sliding condition and convergence condition, respectively. Since the condition (27) itself may cause instability and divergence, the convergence on the sliding surface is assured by the condition (28).

Remark 3: Owing to the discontinuity of the signum function \( \operatorname{sgn}(\cdot) \), chattering may occur in the control input. To alleviate the chattering phenomenon, the boundary layer technique is adopted by replacing the signum function in (20) with the saturation function [18]

\[
\operatorname{sat}(s_k) = \begin{cases} 
\operatorname{sgn}(s_k) & \text{if } |s_k| > \epsilon \\
\frac{s_k}{\epsilon} & \text{if } |s_k| \leq \epsilon 
\end{cases}
\]

where the boundary layer thickness \( \epsilon \) ensures that \( s_k \) is bounded by \( \pm \epsilon \). In practice, a tradeoff between the chattering and tracking error is needed to assign the parameter \( \epsilon \).

4 Experimental studies

In this section, the designed DSMC controller is validated by conducting a series of experimental studies on a prototype system.

4.1 Experimental setup

The experimental setup of a piezoelectric microgripper prototype is depicted in Fig. 2. The gripper is composed of two four-layer piezoelectric bimorph actuators with dimension of \( L \times H \times W = 26 \times 5 \times 0.86 \text{ mm}^3 \) (see Fig. 1). In the current research, one actuator is used which is driven by a high-voltage amplifier (model: EPA-104 from Piezo Systems, Inc.). The end-point position of the actuator is measured by a laser displacement sensor (model: LK-H055, from Keyence Corp.), which has a resolution of 25 nm within a measuring range of 20 mm. In addition, a National Instruments (NI) cRIO-9075 real-time controller (from National Instruments Corp.) equipped with NI-9263 analogue output module and NI-9870 input module is adopted to produce excitation voltage signals and acquire the sensor readings. The NI cRIO-9075 combines a real-time processor and a reconfigurable field-programmable gate array within the same chassis. The chassis is connected to a computer via Ethernet port for communication. Moreover, LabVIEW software is employed to realise a deterministic real-time control of the piezoelectric actuator system.

4.2 Plant model identification

By applying an input sine wave with the amplitude of 0.5 V and varying frequency of 1–1000 Hz to the actuator, the endpoint position responses are recorded. The input–output data sets are then used to identify the plant transfer function by estimating a model \( G_p \) from the frequency response data. The identified second-order model is shown as follows

\[
G_p(s) = \frac{1.141 \times 10^8}{s^2 + 126.8s + 5.943 \times 10^6}
\]

The frequency responses obtained from the experimental data and the identified model \( G_p \) in (30) are compared in Fig. 3.

In the current research, the simple second-order model is employed to demonstrate the effectiveness of the proposed control scheme. By comparing (2) and inverse Laplace transform of (30), the dynamics model parameters can be derived as: \( b = 126.8 \text{ N/s/\mu m} \), \( k = 5.943 \times 10^6 \text{ N/\mu m} \) and \( d = 1.141 \times 10^8 \text{ \mu m/V} \).

The dominant time constant of the plant is calculated as \( \tau = 0.0158 \text{ s} \). Generally, for the digital control implementation, it has been suggested to select a sampling time less than \( \tau/3 \) (i.e. 0.0053 s) of the plant [32]. In the current research, a sampling time is chosen as 0.004 s. With a sampling time \( T = 0.004 \text{ s} \), the discrete-time model parameters are obtained by resorting to (6).

In the literature, the influence of sampling time \( T \) on the performance of sliding-mode control systems has been investigated in [24]. It has been shown that the smaller the sampling time interval, the better the control performance. Hence, a smaller sampling time can be adopted to further improve the control results.
4.3 Experimental results and discussions

For a comparative study, the traditional PID controller is also implemented

\[ u_k^{\text{PID}} = K_p e_k + K_i \sum_{j=0}^{k} e_j + K_d (e_k - e_{k-1}) \]  \hspace{1cm} (31)

where the position error \( e_k = x_{d,k} - x_k \) with \( x_{d,k} \) and \( x_k \) representing the desired and actual system output at the \( k \)th time step, respectively. In addition, \( K_p, K_i \) and \( K_d \) denote the proportional, integral and derivative gains, respectively. In the current research, the control gains are tuned by trial-and-error approach to yield a small tracking error.

4.3.1 Set-point positioning results: First, the set-point positioning capability of the designed controller is examined. The parameters of the DSMC and PID controllers are tuned by trial and error to generate quick response with the same magnitude of overshoot. By selecting \( \lambda_p = 30, \lambda_i = 1, \lambda_d = 1 \times 10^4 \) and \( \epsilon = 100 \), the positioning result of the DSMC #1 is depicted in Fig. 4. In addition, the parameters of PID controller #1 are tuned as \( K_p = 0.017, K_i = 0.534 \) and \( K_d = 0.0002 \). The results of two controllers are compared in Table 1, where the steady-state root-mean-square error (RMSE) is calculated from the last one-second (3–4 s) data.

By inspecting the positioning results as shown in Fig. 4, it is found that both controllers produce no explicit overshoot. Moreover, the DSMC achieves a shorter settling time as well as lower steady-state error, which indicates the improvement of 49.5 and 26.2%, respectively, over the PID control. The control actions as depicted in Fig. 4b shows that no chattering effect exists in DSMC. Besides, further decrease of control gain \( \lambda_p \) or increase of \( \lambda_d \) will produce quicker transient response at the cost of clear overshoot effect. Therefore a compromise between transient speed and overshoot is required to adjust the control gains in practice.

Besides, the time evolutions of the sliding function \( s_k \) and its variation \( \Delta s_k \) for the DSMC are depicted in Fig. 5a. The perturbation estimation error is shown in Fig. 5b which indicates an initial value about \( 1.2 \times 10^5 \). Fig. 5a exhibits that \( \Delta s_k \) is well constrained within the QSD, which confirms the QSD as predicted by (26) for the given parameters and the perturbation estimation error. In addition, in the controller DSMC #1, \( \lambda_d = 1 \times 10^4 \) is selected, which is less than the initial value of \( |\tilde{p}_k| \). Hence, it is not necessary to assign \( \lambda_d \) as the initial value of \( |\tilde{p}_k| \). Actually, the initial value of \( |\tilde{p}_k| \) is not always available in practice. Thus, \( \lambda_d \) is usually assigned by the trial-and-error approach.

In addition, Fig. 5a reveals that when the sliding mode is reached, the sliding function \( s_k \) lies within the interval between \(-15 \) and \(-4 \). Hence, \( s_k \) is well constrained by the boundary thickness parameter \( \epsilon = 100 \). On the other hand, the sliding function variation \( \Delta s_k \) locates between \(-3 \) and \( 3 \). However, it does not arrive at zero because of the inherent property of discrete sliding mode. Even so, \( \Delta s_k \) is more close to zero than \( s_k \). This phenomenon confirms the effectiveness of the selected reaching law (14) in this research.

Moreover, the data points that do not satisfy the condition of \( \lambda_d \geq |\tilde{p}_k| \) in (24) are denoted by circles in Fig. 5b. The time evolutions of the sliding condition (27) and the convergence condition (28) are shown in Figs. 5c and d, respectively. Fig. 5d illustrates that all of the circle points meet the convergence condition except for the initial one at the time of \( 1 \) s. On the other hand, this initial point satisfies the sliding condition as revealed in Fig. 5c. This indicates that the convergence on the sliding manifold is reached in one sampling period. Hence, all of the points meet either the sliding condition or the convergence condition. This explain the reason why the control system still converges even though the relationship of \( \lambda_d \geq |\tilde{p}_k| \) is not met.

4.3.2 Sinusoidal tracking results: Next, the tracking performance of the designed controller for a 1.25-Hz sinusoidal motion (see Fig. 6a) is verified. Although the motion tracking can be implemented by employing the foregoing
PI#1 and DSMC #1 controllers directly, relatively large tracking errors are produced. In order to achieve better tracking accuracy for the sinusoidal input, PID #2 and DSMC #2 controllers are adopted by finely tuning the parameters through several trials. Specifically, PID #2 is tuned manually as $K_p = 0.032$, $K_i = 1.011$ and $K_d = 8 \times 10^{-5}$. DSMC #2 parameters are adjusted as $\lambda_P = 0.5$, $\lambda_I = 1$, $\lambda_S = 8 \times 10^5$ and $\epsilon = 10$. The positioning errors of PID #2 and DSMC #2 are shown in Fig. 6b.

For comparison, one-cycle positioning results (2.4–3.2 s) of both controllers are tabulated in Table 1, where the percent maximum error (MAXE) and RMSE are calculated as follows

$$\text{MAXE}\% = \frac{\max(|e_k|)}{\max(x_{d,k}) - \min(x_{d,k})} \times 100\% \quad (32)$$

$$\text{RMSE}\% = \frac{\sqrt{\frac{1}{N} \sum_{k=1}^{N} e_k^2}}{\max(x_{d,k}) - \min(x_{d,k})} \times 100\% \quad (33)$$

It is observed that the DSMC produces smaller tracking errors than PID controller. Specifically, the DSMC approach achieves the MAXE and RMSE of 0.118 and 0.049 μm,
respectively, which are almost the noise level of the sensor. These results indicate significant enhancement of 79.6 and 84.5% in comparison with the PID results, respectively.

In addition, Figs. 7a and b depict the time history of \( s_k \) as well as \( \Delta s_k \) and the perturbation estimation error \( \tilde{p}_k \) of DSMC #2, respectively. It is observed from Fig. 7a that \( \Delta s_k \) is well restricted within QSD band. By selecting a larger value of \( \lambda_S = 8 \times 10^5 \), \( |\tilde{p}_k| \leq \lambda_S \) holds for this motion trajectory except for one point at the time of 0.8 s (the initial point of the sinusoidal trajectory). In addition, both \( s_k \) and \( \Delta s_k \) vary in the vicinity of zero as shown in Fig. 7a. Moreover, \( \Delta s_k \) is one order of magnitude lower than \( s_k \), which also demonstrates the effectiveness of the adopted reaching law (14).

The sliding condition and convergence condition of the discrete sliding mode are depicted in Figs. 7c and d, respectively. It is seen that the point at 0.8 s meets the sliding condition as denoted by a circle in Fig. 7c. In addition, almost all the data points satisfy the convergence condition except for four points as shown in Fig. 7d (diamond markers). Fig. 7c reveals that these four points meet the sliding condition instead. That is, the controller converges since all of the points in the trajectory satisfy either sliding or convergence condition.

4.3.3 Arbitrary motion tracking results: Then, the tracking performance of the proposed control scheme for an arbitrary reference input is tested. Particularly, by applying an arbitrary input as shown in Fig. 8a, the tracking errors of the aforementioned PID #2 and DSMC #2 controllers are shown in Fig. 8b. The errors are quantified in Table 1. As compared with the PID results, the DSMC has reduced the MAXE and RMSE by 84.8 and 87.1%, respectively. Hence, the superiority of DSMC over PID is evident from the experimental results.

For this arbitrary trajectory, the sliding function and perturbation estimation error of the DSMC are shown in Figs. 9a and b, respectively. Similarly, Fig. 9a reveals that \( \Delta s_k \) is well constrained by the width of QSD. It is seen that a large estimation error of \( \tilde{p}_k = 3.2 \times 10^6 \) occurs at the time of 1 s. This indicates a sudden change of \( \dot{p}(t) \), which is mainly caused by the sharp transition of \( \dot{x}(t) \) at 1 s.

In addition, the assigned \( \lambda_S = 8 \times 10^5 \) is smaller than the maximum value of \( |\tilde{p}_k| \) as shown in Fig. 9b. The data points that violate the condition of \( |\tilde{p}_k| \leq \lambda_S \) are denoted as circles in Fig. 9b. For these points, the corresponding sliding and convergence conditions are marked as circles in Figs. 9c and d, respectively. Fig. 9d illustrates that almost all the points satisfy the convergence condition except for a few points which are denoted by diamond markers. Alternatively, these points meet the sliding condition as represented in Fig. 9c. Therefore the closed-loop system still converges since the data points that violate the condition of \( |\tilde{p}_k| \leq \lambda_S \) satisfy either sliding or convergence condition of the discrete sliding mode. The experimental results reveal that the relationship (24) is not a necessary condition for the existence of discrete-sliding mode.

4.3.4 Robustness testing results: The foregoing experiments confirm the robustness of the presented control
with respect to internal disturbances because of model uncertainties and parameter perturbations. The robustness against external disturbance is also examined by applying an external force on the piezoelectric actuator during a motion tracking task.

Specifically, the external force is applied by hanging a weight of 25 mN on the piezoelectric bimorph during the sinusoidal motion tracking. By exerting the force as shown in Fig. 10a, the tracking result of DSMC #2 is described in Fig. 10b. In addition, the time evolutions of $s_k$ and $\Delta s_k$ are plotted in Fig. 10c, and the tracking errors are depicted in Fig. 10d. Alternatively, by applying the same external force, the tracking result of the PID #2 controller is depicted in Fig. 11a, and the tracking error is shown in Fig. 11b.

It is observed that the external force causes an increase of the tracking error at the moment of occurrence (around 1.7 s) for both PID and DSMC control schemes. As compared with the maximum error of 27.67 $\mu$m produced by PID controller, the DSMC is able to suppress the maximum error to 4.75 $\mu$m. That is, DSMC has mitigated the maximum error by 83% as compared with PID result. Moreover, after the
occurrence of the external disturbance, the tracking accuracy of DSMC is recovered quickly within 0.18 s, whereas the result of PID is restored after 0.30 s. Thus, as compared with PID, DSMC is capable of shortening the recovery time by 40%. These results demonstrate that the DSMC possesses a much better robustness property than PID against external disturbances.

4.4 Discussions

The experimental investigations confirm the effectiveness of the proposed DSMC control scheme with Assumption 1. Actually, the perturbation estimation errors $\tilde{p}_k$ as plotted in Figs. 5b, 7b and 9b are obtained by (9), that is, $\tilde{p}_k = p_{k-1} - p_k$. Experimental results also reveal the superiority of the DSMC over PID strategy in terms of transient-state response time and steady-state positioning error. The reason why the finely tuned PID controller is still not capable of achieving satisfactory positioning results mainly attributes to the non-linear hysteresis effect of the piezoelectric actuator. In contrast, the fact that a rapid and precise positioning is accomplished by the DSMC strategy demonstrates the effectiveness of the proposed control scheme without modelling the hysteresis effect.
It is seen from Fig. 3 that the second-order model matches the magnitude response of system well at low frequencies up to 500 Hz. However, an explicit phase error starts already at 20 Hz. To capture the dynamics behaviour accurately, a higher-order model is required to be identified. In the current research, a simple second-order model is employed and the higher-frequency dynamics is involved in the lumped disturbance $p_s$. The achieved experimental results demonstrate the effectiveness of the proposed control scheme using a lower-order plant model.

The experimental results illustrate that the relationship (24) is a sufficient but not necessary condition for the existence of discrete sliding mode. A necessary and sufficient condition is expected to be deduced in the next step. In addition, only a local stability of the closed-loop system is proved in this research. The issue of deriving a simple control scheme with global asymptotic stability is a topic of ongoing work. Besides, in the future, the adaptive mechanism [10, 11, 33] will be employed to adjust the controller parameters automatically.

In addition, the quantisation errors were not considered in the discretisation approach in the current research. In the literature [25], it has been derived that the quantisation errors will increase the width of QSD. In the future, the influence of the quantisation errors on the performance of the presented control will be investigated for potential applications using 8- or 16-bit microcontrollers.

5 Conclusions

This paper is dedicated to precise motion control of a piezoelectric bimorph actuator. Instead of using a state observer, a new DSMC scheme has been developed based on a second-order plant model of the system. The unmodelled non-linearity effect is considered as a lumped perturbation which is estimated by resorting to one-step delayed estimation technique. The effectiveness of the presented approach without using state observer and hysteresis model has been validated by experimental investigations. Both setpoint positioning and sinusoidal tracking results confirmed the superiority of the DSMC over PID control in terms of response speed, positioning accuracy as well as robustness property. The presented idea can be extended to precise control of other systems that can be described using a discrete-time plant model preceded by disturbances.

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7 References


