Design and Robust Repetitive Control of a New Parallel-Kinematic XY Piezostage for Micro/Nanomanipulation

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Abstract—This paper presents mechanism and controller design procedures of a new piezoactuated flexure XY stage for micro-/nanomanipulation applications. The uniqueness of the proposed stage lies in that it possesses an integrated parallel, decoupled, and stacked kinematical structure, which owns such properties as identical dynamic behaviors in X and Y axes, decoupled input and output motion, single-input-single-output (SISO) control, high accuracy, and compact size. Finite element analysis (FEA) was conducted to predict static performance of the stage. An XY stage prototype was fabricated by wire electrical discharge machining (EDM) process from the alloy material Al7075. Based on the identified plant transfer function of the micropositioning system, an \( H_{\infty} \) robust control combined with a repetitive control (RC) was adopted to compensate for the unmodeled piezoelectric non-linearity. The necessity of using such a combined control is also investigated. Experimental results demonstrate that the \( H_{\infty} \) plus RC scheme improves the tracking response by 67% and 28% compared to the stand-alone \( H_{\infty} \) for 1-D and 2-D periodic positioning tasks, respectively. Thus, the results illustrate the effectiveness of the proposed mechanism design and control approach.

Index Terms—Flexure mechanisms, micro-/nanopositioning, motion control, parallel manipulators, piezoelectric actuation.

I. INTRODUCTION

A MICROPOSITIONING system is a crucial component for the robust micro-/nanomanipulation system dedicated to automated ultra-high-precision positioning and assembly in micro-/nanometer scales, such as bio-cell manipulation [1], optical fiber alignment [2], and scanning probe microscopy (SPM) [3]. Such kind of applications call for micropositioning systems capable of positioning with high resolution, high repeatability, and high bandwidth. Although micromanipulation is an existing topic in both academia and industry, this technique has gained extensive recent attentions in the research domain. The realization of a high-performance manipulation is a challenging work because it requires an integrated consideration of all the technical issues including mechanical joints, actuators, sensors, kinematic schemes, materials, fabrication processes, control strategies and so on.

In the literature, flexure-based compliant stages [4], [5] are popularly recognized to achieve ultrahigh-precision positioning. The reason lies in that the compliant mechanisms deliver motions by making use of the elastic deformations of notch hinges instead of conventional mechanical joints, which renders the stage with merits of free of backlash, zero friction, repeatable motion, and vacuum compatibility. Besides, piezoelectric actuators (PZTs) [6]–[8], are frequently used for the actuation since they are capable of positioning with (sub)nanometer level resolution, large blocking force, high stiffness, and rapid response characteristics. Some piezostages have been developed in research laboratories and even commercialized on the markets (e.g., the piezostages manufactured by Physik Instrumente GmbH). This research investigates XY stages due to their promising applications in micro-/nanomanipulation and positioning.

According to kinematic schemes, the XY stages can be classified into two categories in terms of serial and parallel ones [9]. In serial stages, two one-degree-of-freedom (1-DOF) linear stages are jointed consecutively as a stacked or nested architecture in majority of the cases [10]–[13], where the moving (output) platform is only supported by the last stage. The serial stage has the advantages of simple structure and control strategy since the two stages can be treated independently. However, it exhibits certain disadvantages including high inertia, asymmetric dynamics in X and Y axes, and cumulative errors. In contrast, parallel stages are based on parallel kinematics, whose moving platforms are supported by all the connecting limbs simultaneously [4], [14], [15]–[18]. By this way, the performances of low inertia, high load-carrying capacity, identical dynamic behaviors in X and Y axes, and high accuracy can be relatively easily achieved. Therefore, parallel stages are more appropriate for micro-/nanopositioning from the above considerations.

This research is concerned with macro-scale parallel stages with the size of tens to hundreds of millimeters, although the stages can also be fabricated in micro- and mesoscales in accordance with the pertinent application requirements [19]. Within macro-scale, the existing parallel XY stages [4], [14], [15]–[18], are all designed as planar monolithic structures, which are free of assembly and can be easily manufactured by such process as wire electrical discharge machining (EDM). In consequence, the monolithic stage potentially occupies a large planar dimension since it is fabricated from a piece of material. Under such
situations, it may not be suitable for the applications where the micro-/nanomanipulation inside a limited space, e.g., inside a scanning electron microscope [20], is required.

One motivation of the current research is to overcome the shortcomings of monolithic XY parallel stages. Specifically, a concept of parallel stage with stacked structure is presented to obtain a compact size. By this way, the inherent advantages of parallel kinematics are maintained. Moreover, under the circumstances where the stage is underactuated or sensory feedback of the output motion is not permitted, a decoupled XY stage with proper calibration is desirable [16]. In order to isolate/protect actuators and to obtain a decoupled output motion, it has been recommended in [17] that a totally decoupled stage is preferred to be designed with both input and output decoupling properties. In this paper, a parallel piezostage with both stacked and totally decoupled structure is proposed for micro-/nanomanipulation applications.

The major problem of a piezoactuated system arises from the nonlinearity due to hysteresis and creep effects. In order to fulfill the requirements of ultrahigh-precision positioning, the hysteresis has to be suppressed by an appropriate control scheme. Generally, the existing schemes fall into two categories, i.e., hysteresis model-based and hysteresis model-free methods. In the first category of approaches, a hysteresis model (e.g., Preisach model) is generated and used to construct an inversion-based feedforward compensator [21]–[23]. It has been shown that the inversion-based compensation can achieve an accurate positioning, whereas the result is very sensitive to the model accuracy [3], [24]. Fortunately, the combination of feedforward with feedback control can be adopted to suppress the hysteresis as well as creep effects [25], [26]. Concerning the second category, its main advantage lies in that no hysteresis model is required. The unmodeled hysteresis is considered as an uncertainty or a disturbance [27] to the nominal system, which is tolerated by a robust or adaptive controller. For instance, the applications of sliding mode control [28], \( H_\infty \) robust control [29], fuzzy logic control [30], and neural network control [31] have been reported. In addition, \( H_\infty \) combined with iterative learning control is applied to nanopositioning systems [32], and the integration of inversion-based feedforward and \( H_\infty \) control is realized in PZTs [33]. It is noticeable that another method named charge actuation can be used to reduce hysteretic behavior of PZT. If a PZT actuator is driven by a charge source, the hysteresis becomes almost negligible. This technique has been known for some time, and there has been renewed interest in this subject recently [34].

In this paper, a robust repetitive control is adopted to suppress the unmodeled hysteresis and to alleviate tracking errors for periodic reference input. More specifically, a 2-DOF control framework employing an \( H_\infty \) robust control combined with a plug-in repetitive controller is used to suppress the nonlinearity in the piezo-driven positioning system without modeling the hysteresis and creep effects. In the literature, although both \( H_\infty \) control [29], [32], [33] and repetitive control (RC) [35]–[38] have been well studied, the investigation on their combination is still limited [39], [40]–[42]. It remains unclear whether such a combined controller is necessary for piezo-driven systems which suffer from hysteretic and creep nonlinearities. In this paper, the necessity to introduce such a combination for a micropositioning system is presented in detail by revealing: 1) the superior performance of \( H_\infty \) over traditional proportional-integral-derivative (PID) control, 2) the better performance of \( H_\infty \) plus RC over PID plus RC, and 3) the better performance of \( H_\infty \) combined with RC over stand-alone \( H_\infty \) and RC approaches.

In the rest of the paper, the mechanical design procedures of an XY stage are described in Section II, where the stage performance is tested via finite element analysis (FEA). In Section III, a prototype XY stage is fabricated along with plant model identified. Then, an \( H_\infty \) combined with repetitive control is presented in Section IV to improve the tracking ability of the positioning system. The performance of the piezostage system is verified by experiments conducted in Section V. Finally, Section VI concludes this paper.

II. MECHANISM DESIGN AND FINITE ELEMENT ANALYSIS

A. Mechanism Design and Assembly Process

To design a decoupled XY stage with parallel structure, a 2-PP (P stands for prismatic joint) mechanism is adopted due to its simple structure. In what follows, the design procedures of a PP limb with decoupled translations are outlined.

It is known that the compound parallelogram flexure [see Fig. 1(a)] provides 1-DOF ideal translation (\( d_x \)) if a force \( F_z \) is applied on its output stage. At the same time, the compound bridge-type displacement amplifier [see Fig. 1(b)] delivers both an amplified translational output motion in vertical direction (y-axis) and a spring preload for the actuator. Besides, this type of amplifier has much larger input stiffness and larger lateral stiffness than the conventional bridge-type amplifier [43]. The large input stiffness calls for an actuator with large blocking force and stiffness. Hence, PZT is the most suitable actuator for driving the amplifier. On the other hand, the large lateral stiffness indicates that the output end of the amplifier can tolerate a large lateral load. This merit offers protection for the inner PZT which can only tolerate small magnitude of lateral load. Therefore, combining the preceding 1-DOF stage with the amplifier, a PP limb with decoupled translations along the x- and y-axes is selected as shown in Fig. 1(c). This PP limb is employed as a basic module to design an XY parallel stage with decoupled structure.
With different arrangement schemes of two such limbs, various architectures of an monolithic XY stage can be designed. Furthermore, four PP limbs can be adopted to create a double-symmetric architecture. For instance, a 4-PP stage is developed in previous works [44] of the authors. As monolithic structures, the XY stages are easy to fabricate. However, they own a relatively large dimension in plane. Hence, they are not suitable for the applications where the micro-/nanomanipulation inside a limited space is required. Although the stage structures can be further enhanced to utilize the planar space efficiently to get a more compact 2-D structure, it still occupies a large area if long-stroke PZT actuators are employed to obtain a large workspace.

To conquer such drawbacks, an XY stage with a two-layer stacked structure is proposed as illustrated in Fig. 2. Using an orthogonal assembly of the two PP limbs, the stage produces decoupled output motion. While comparing to monolithic XY stages, it has a more compact size. Its limitation lies in that, assembly is required to tie the two limbs and output platform together to construct the whole stage. Considering that the fixing holes can be easily machined with a fine tolerance (e.g., ±5 μm), precise assembly of the components is not impossible nowadays. The induced slight squareness errors can be tolerated by the flexures suffering from elastic deformations. Although four PP limbs can be employed to design a double-symmetric XY stage to reduce parasitic motions and thermal gradient effects, the two-limb version as shown in Fig. 2 is adopted in this paper to demonstrate the conceptual design of a compact XY stage with integrated parallel, decoupled and stacked structure.

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The exploded view of a computer-aided design (CAD) model for such an XY stage is graphically shown in Fig. 3(a). The assembly is completed through the following processes.

1) First, limb 1 (with PZT 1 embedded) is fixed on the base through two bolts, where spacer 1 is used to support the limb 1.
2) Second, the output platform is fixed at limb 1 through two bolts.
3) Third, limb 2 is placed on the top of limb 1 separated by spacer 2, and then mounted on the base by three bolts.
4) Forth, the output platform is assembled on limb 2 via another two bolts.
5) Afterwards, the top platform is mounted on the top of the output platform in order to support external weights.
6) Finally, two displacement sensors are fixed on the base for the measurement of the output platform positions.

Fig. 3. (a) Exploded view of CAD model for the XY stage. (b) Assembled model of the XY stage.

The exploded view of a two-layer XY stage with parallel, decoupled, and stacked structure.

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The assembled model is illustrated in Fig. 3(b). It is noticeable that although the right-circular shape flexure hinge [see Fig. 4(a)] is adopted in the current research for illustrations, other types of hinges can also be employed.

### B. Performance Evaluation with FEA

Finite element analysis (FEA) is performed with ANSYS to predict the performance of the designed XY stage.

The architectural parameters of the stage [see Fig. 1(a) and Fig. 4] are tabulated in Table I, and the physical and mechanical parameters of the alloy Al7075 used in the FEA are: Young’s modulus = 71.7 GPa, yield strength = 503 MPa, Poisson’s ratio = 0.33, and density = $2.81 \times 10^3$ kg·m$^{-3}$. A 3-D finite element model is established with 20-node element SOLID186. The mesh model is created with a medium mesh size and refined at the flexure hinges to obtain smaller mesh size therein in order to achieve more accurate simulation results. Besides, zero displacements are assigned on the surfaces of the mounting holes to constrain the mechanism.

In the static structural FEA simulation, a force is applied on the input ends of the amplifier 2 (in limb 2), which produces...
In this section, the development of a prototype XY parallel micropositioning stage is described and the plant model is identified for subsequent studies.

A. Prototype Fabrication and Experimental Setup

One prototype of the XY stage is fabricated as shown in Fig. 6. As far as materials are concerned, Al7075 alloy is more elastic and has a lower density compared to steel. Thus, Al7075 plates are used to fabricate the two monolithic limbs of the XY stage, which are then assembled together as per the processes depicted in Fig. 3. The assembled XY stage has an overall dimension within $116 \times 116 \times 46$ mm$^3$. Regarding to actuation, the actuators should be selected with high enough stiffness to drive the stage which has an input stiffness (6.99 N/μm) as evaluated by FEA in Section II-B. Two 20 μm-stroke PZTs (model PAS020 with stiffness of 14–208 N/μm ± 20%, from Thorlabs, Inc.) are adopted in this research. In order to measure the output displacements of the moving platform, two laser displacement sensors (Microtrak II model LTC-025-02, from MTI Instruments, Inc.) are used. Concerning control apparatus, a dSPACE DS1005 (from dSPACE GmbH) rapid prototyping system equipped with DS2001 A/D and DS2102 D/A modular boards are employed. The D/A board produces an analog voltage which is then amplified by a two-axis voltage amplifier (BPC002 from the Thorlabs) to drive the PZTs. Besides, the sensor outputs are passed through signal conditioners and then acquired simultaneously by using the A/D board. The overall experimental setup is sketched in Fig. 6.

Using a signal of 0–10 V provided by the D/A board, the voltage amplifier generates an output signal of 0–75 V to drive the PZT actuator, which produces a workspace range of $138 \times 128$ μm$^2$ for the XY stage.

B. Plant Model Identification

The system plant consists of the PZTs, XY stage, and laser sensors. Since it is difficult to obtain an accurate physical model of the whole system, a linear model of the plant is experimentally identified with respect to a specific operating point. The identified model is called a nominal model for the system, and the uncertainty (including hysteresis and creep effects) is expected to be compensated by implementing a robust controller.

Swept sine waves with the frequencies covering the bandwidth of interest (1–600 Hz in the current research) are created with “Chirp” block in Simulink and applied to voltage amplifier to drive the PZTs. When the input signal has a large amplitude (compared to the input voltage range of 10 V), piezoelectric hysteresis effects become significant. In contrast, the plant model can be assumed to be linear over a short operating range [29]. Thus, to obtain a linear model of the system, a low voltage of 0.5 V (peak-to-peak amplitude), which only accounts for 5% of the total input range, is selected to reduce the hysteresis effect. With such a small input signal, the modeling error caused by hysteresis can be neglected.

To identify the model, PZT 1 is first actuated with the swept sine-wave voltage. At the same time, the displacements of the output platform in both x- and y-axes are measured and recorded with a sampling rate of 2 kHz. To reduce the noise effect, the data collection procedure is experimentally repeated five times and the five sets of output data are averaged as the sensor output. With the two PZTs actuated individually, the frequency responses (see Fig. 7) in the two axes are obtained by fast Fourier transform (FFT). The frequency response of the i-axis motion induced by driving the PZT $k$ is described by $G_{ik}(j\omega) = d_i(j\omega)/u_k(j\omega)$, where the axis index $i = x$ and $y$, and actuator index $k = 1$ and 2, respectively. It is noticed that, with PZT 1 driven, the responses in the y-axis are 22 dB
Fig. 7. Magnitudes of frequency responses of the two-input two-output micropositioning stage. The input is the voltage applied to PZT, and output is the displacement measured by laser sensor. $G_{x1}$ represents the x-axis frequency response when the stage is driven by PZT 1.

lower than that in the x-axis in low-frequency range except for the frequencies nearby the resonant mode. With PZT 2 driven individually, similar results are obtained. Thus, the two axial motions of the XY stage are well decoupled, and two single-input-single-output (SISO) controllers can be designed for each axis. In comparison with the FEA result which shows a crosstalk of 0.5%, the experimental result (7.9%) is more significant. The discrepancy may come from the machining error and the installation error of the stage. In addition, the misalignment of the laser sensors with respect to the target platform and the perpendicular error of the two sensors lead to non-orthogonality of the metrology coordinate frame and also result in crosstalk between the two working axes.

The input-output data sets are used to identify the plant transfer functions ($G_{x1}$ and $G_{y2}$) in the two axes by resorting to the command “spafdr” in Matlab, which estimates the model from the frequency responses. For instance, the frequency response of the x-axis plant model $G_{x1}$ identified at the null position (0 V initial input for PZT) is displayed in Fig. 8. It is observed that there are several resonance modes in the plant. The one with the most significant magnitude occurs at 183 Hz. A 16th-order nominal model $G_{xn}$ with the unit of $\mu$m/V is identified in continuous-time form as shown in (1), at the bottom of this page, which matches the system dynamics well in the range of 1 to 600 Hz. The response of the corresponding discrete-time model generated using a sampling rate of 2 kHz is also depicted in Fig. 8. It is observed that the discrete-time model approximates the continuous-time one quite well using the selected sampling rate.

Besides, the system frequency responses are generated at different operating points (0 to 4 V) for the PZT. Comparing the frequency responses as plotted in Fig. 9, one can observe that the responses vary as the changing of the operating point.

$$G_{xn}(s) = \frac{-675.1937(s - 7787)(s^2 + 1.225s + 4.127 \times 10^5)(s^2 + 39.91s + 1.075 \times 10^6)}{(s^2 + 34.074s + 3.721 \times 10^5)(s^2 + 53.84s + 9.283 \times 10^5)(s^2 + 23.48s + 1.307 \times 10^6)}$$
$$\times \frac{(s^2 - 28.21s + 4.329 \times 10^6)(s^2 - 3175s + 7.989 \times 10^6)(s^2 + 66.58s + 1.002 \times 10^7)}{(s^2 + 2318s + 3.646 \times 10^6)(s^2 + 23.07s + 4.831 \times 10^6)(s^2 + 52.09s + 1.003 \times 10^7)}$$
$$\times \frac{(s^2 + 34.5s + 1.207 \times 10^7)(s^2 + 93.46s + 1.317 \times 10^7)}{(s^2 + 16.26s + 1.203 \times 10^7)(s^2 + 82.15s + 1.313 \times 10^7)}$$

(1)
In addition to the nonlinear hysteresis effect, the plant model variation also necessitates the design of a robust controller.

IV. ROBUST REPETITIVE CONTROLLER DESIGN

The uncertainties in terms of model incompleteness and model variation are the essential of piezo-driven system attributed to the hysteresis and other nonlinearities. To compensate for the uncertainty, an $H_{\infty}$-based robust control scheme is designed in this section.

A. $H_{\infty}$ Robust Control Design

In comparison with other feedback control approaches, the mixed-sensitivity $H_{\infty}$ robust control exhibits the advantage of combining the performance and robustness requirements in one controller design procedure. The achievement of the performance and robustness for $H_{\infty}$ controller is heavily dependent on the selection of the weighting functions.

The block diagram of the closed-loop $H_{\infty}$ robust control is shown in Fig. 10, where $W_1$, $W_2$, and $W_3$ represent the weighting functions. $z_1$, $z_2$, and $z_3$ are weighted signals. The goal of $H_{\infty}$ controller design is to minimize the transfer functions of these weighted signals. The sensitivity function $S$ and complementary sensitivity function $T$ of the closed-loop system can be obtained as $S = 1/(1 + G_x G_c)$ and $T = 1 - S$, where $G_x$ is the plant model and $G_c$ denotes the $H_{\infty}$ controller to be designed. In order to design the controller, the three weighting functions ($W_1$, $W_2$, and $W_3$) need to be determined to shape the sensitivity function $S$, transfer function $G_c S$, and complementary sensitivity function $T$, respectively. With reference to Fig. 10, we can deduce that $S$ denotes the transfer function between the reference input $x_d$ and tracking error $e$, $G_c S$ relates $x_d$ to the control input $u$, and $T$ relates $x_d$ to the sensor output $x$ as well as the sensor noise $n$ to sensor output $x$.

In light of the small-gain theorem, the controller $G_c$ can be synthesized by minimizing the transfer functions of the weighted signals, i.e.,

$$\|N\|_{\infty} = \sup_{\omega \in \mathbb{R}} \sigma[N(j\omega)] \leq 1; \quad N = \begin{bmatrix} W_1 S \\ W_2 G_c S \\ W_3 T \end{bmatrix}$$

where $\| \cdot \|_{\infty}$ denotes the $H_{\infty}$ norm to measure the size of a transfer function, which is considered as the maximum singular value $\sigma[N(j\omega)]$ of the transfer function with the argument $\omega$ denoting the frequency. Such a controller guarantees that $\|W_1 S\|_{\infty} \leq 1$, $\|W_2 G_c S\|_{\infty} \leq 1$, and $\|W_3 T\|_{\infty} \leq 1$. It follows that the inverse of the three weighting functions provide upper bounds on the corresponding transfer functions (see Fig. 11). Therefore, the key technique of designing an $H_{\infty}$ robust controller lies in the selection of the weighting functions which are stated below.

The weighting function $W_1$ is desired to exhibit high gains at low frequencies and low gains at high frequencies. This ensures that the sensitivity function is small at low frequencies in order to guarantee a small tracking error within the bandwidth of interest. In contrast, the weighting function $W_3$ is chosen such that it has low gains at low frequencies and high gains at high frequencies. This strategy is taken to ensure that the complementary sensitivity function rolls off at high frequencies to attenuate the sensor noise and hence to produce a better resolution. Besides, the weighting function $W_2$ is chosen as a constant to make the control input of PZT stay below the saturation limit.

In this research, the weighting function $W_1$ is designed as a first-order low-pass filter in the form

$$W_1(s) = \frac{0.1667s + 188.5}{s + 0.01885}$$

which is chosen to make the sensitivity function $S$ have a gain less than $-80$ dB at low frequencies with a $-3$ dB crossover frequency around $30$ Hz and thereby insensitive to low frequency variations in the nominal model.

The weighting function $W_3$ is selected as a first-order high-pass filter transfer function

$$W_3(s) = \frac{s + 37.7}{0.001s + 377}$$

which is assigned to enable the complementary sensitivity function $T$ to have a gain less than $-60$ dB at high frequencies with a roll-off slope of $-20$ dB/decade.
Besides, the weighting function for control input scaling is chosen as a constant $W_2 = 0.5$ in order to restrict the control signal within the upper bound of 10 V.

Using the selected weighting functions, an optimal $H_\infty$ robust controller is designed based on the $\gamma$-iteration method (using the function “hinfsyn” in Robust Control Toolbox of Matlab). The obtained controller is a 18th-order model, which is then converted to a discrete-time form using zero-order hold (ZOH) with the sampling time interval of 0.5 ms (using the function “c2d” in Matlab). The −3 dB crossover frequencies of the sensitivity function and complementary sensitivity function give a bandwidth measurement of the positioning system. From the singular value plots of $S$ and $T$, as shown in Fig. 11, it is observed that the corresponding crossover frequencies are 29.8 Hz and 49.0 Hz, respectively. A small peak of 1.96 dB in the measurement of the positioning system. From the singular value plots of $S$ and $T$, as shown in Fig. 11, it is observed that the corresponding crossover frequencies are 29.8 Hz and 49.0 Hz, respectively. A small peak of 1.96 dB in

\[
\tilde{G}(z) = \frac{G(z)z^{-N}}{1 - Q(z)z^{-N}}.
\]
function, of the whole system can be generated as follows.

\[
T_c(z) = \frac{E(z)}{X_d(z) - D(z)} = \frac{1}{1 + [1 + G_c(z)]G_e(z)G_{xn}(z)} \left[ 1 - z^{-N}Q(z) \right]_{z = e^{j\omega T_c}} (10)
\]

where \( E(z) \), \( X_d(z) \), and \( D(z) \) denote the z-transform of the tracking error \( e \), desired position \( x_d \), and external disturbance \( d \), respectively. The last expression in (10) means that the system can be expressed as three cascaded subsystems [46]. The first one \( [1 - z^{-N}Q(z)] \) is a filter with a time delay, which is always stable with \( Q(z) \) selected as a finite impulse response (FIR) filter. The second subsystem \( \frac{1}{1 + G_c(z)G_e(z)G_{xn}(z)} \) has the same denominator as the complementary sensitivity function (5) of the system, which is also stable with the desired feedback controller \( G_c(z) \). Additionally, the third one can be regarded as a closed-loop system with the term \( \frac{1}{1 - k_cG_f(z)G_e(z)G_{xn}(z)} \) \( z^{-N}Q(z) \) in the positive feedback path. In view of the small-gain theory, this subsystem is stable if the following condition is satisfied:

\[
\frac{1}{1 - k_cG_f(z)G_e(z)G_{xn}(z)} \left| Q(z) \right|_{z = e^{j\omega T_c}} < 1 \quad (11)
\]

with \( \omega \in \left[ 0, \frac{\pi}{T_c} \right] \). It can be deduced that the condition (11) is satisfied if the following two conditions hold:

\[
\left| Q(z) \right|_{z = e^{j\omega T_c}} < 1 \quad (12)
\]

\[
\left| 1 - k_cG_f(z)G_e(z)G_{xn}(z) \right|_{z = e^{j\omega T_c}} < 1 \quad (13)
\]

for \( \omega \in \left[ 0, \frac{\pi}{T_c} \right] \).

The condition (12) imposes a constraint on the design of low-pass filter \( Q(z) \), and condition (13) provides a guidance for the selection of control gain \( k_c \).

Furthermore, inserting (5), (6), and (7) into (13), gives

\[
1 - k_cG_f(z)G_e(z)G_{xn}(z) \left| \frac{B_n(z)}{b} \right|_{z = e^{j\omega T_c}} < 1 \quad (14)
\]

Then, taking into account (8) and (14), allows the derivation

\[
0 < k_c < 2 \quad (15)
\]

Therefore, the overall system is asymptotically stable once the above conditions (12) and (15) are satisfied at the same time. It is noticeable that the repetitive controller can also be designed with other selection of \( G_f(z) \). For instance, with \( G_f(z) \) assigned as a phase lead compensator, a digital repetitive controller is constructed in [47] for SPM applications, where a phase stability condition is derived to design \( G_f(z) \).

Concerning the implementation of the repetitive controller, the low-pass filter \( Q(z) \) is usually selected as a finite impulse response (FIR) filter (e.g., \( Q(z) = \frac{1}{2}z + \frac{1}{2} + \frac{1}{2}z^{-1} \)), which has zero phase shift. This filter enables the closed-loop system to have unitary gain at the disturbance fundamental frequency, which guarantees the disturbance rejection of the system. Unfortunately, it slightly moves the pole positions of the closed-loop system in z-plane [46]. Hence, in the current research, \( Q(z) \) is designed alternatively as the discrete-time realization of a first-order filter \( Q(s) = \frac{1}{1 + T_q s} \) with ZOH, i.e.,

\[
Q(z) = Z \left[ 1 - e^{-T_q z} \right] = \frac{1 - e^{-T_q z}}{z - e^{-\frac{T_q}{T}}}, \quad (16)
\]

where \( Z[\cdot] \) denotes the z-transform operator and \( T_q \) is the time constant of the filter \( Q \). It is observed that the designed low-pass filter \( Q(z) \) satisfies the stability condition (12). The effects of the two parameters \( k_c \), and \( T_q \) on control performance are investigated by the following simulation studies.

C. Simulation Studies

1) \( H_\infty \) Robust Controller: First, a simulation study is carried out to demonstrate the tracking performance of the designed \( H_\infty \) controller compared with traditional PID control. A digital PID controller can be expressed by

\[
G_{pid}(z) = K_p + \frac{K_i z}{z - 1} + \frac{K_d(z - 1)}{z} \quad (17)
\]

The controller parameters \( K_p \), \( K_i \), and \( K_d \) are initially tuned using the Ziegler-Nichols method and then finely regulated by trial-and-error to \( K_p = 0.0045 \), \( K_i = 6.8538 \), and \( K_d = 1.9924 \times 10^{-6} \), which give a more rapid response mainly attributed to the integral control efforts as displayed in Fig. 14(a).

For a low-frequency sinusoidal reference input (1 Hz) as shown in Fig. 13(b), the tracking errors of PID and \( H_\infty \) controllers are plotted in Fig. 13(c). Compared to the PID tracking results, the \( H_\infty \) controller reduces the maximum tracking error by 68.2%. Thus, a better performance of \( H_\infty \) over PID control strategy is evident.

2) Robust Repetitive Controller: To demonstrate the effects of the control gain \( k_r \) [see (16)] on the tracking results, simulation studies are carried out below. For a 2-Hz periodic sinusoidal reference input as shown in Fig. 14(a), the tracking errors of \( H_\infty \) plus RC along with different values of \( k_r \) are compared in Fig. 14(b). The variation tendency of the control errors reveals that \( k_r = 1 \) leads to the quickest convergence with a low tracking error. Thus, \( k_r = 1 \) is chosen in the following experimental studies. It is noticeable that for nonperiodic reference input, \( k_r = 0 \) can be assigned in the robust repetitive controller.

Moreover, the robust repetitive controller with a low-pass zero-phase FIR filter \( Q(z) = \frac{1}{2}z + \frac{1}{2} + \frac{1}{2}z^{-1} \) is also implemented, and the simulation results with a longer time (8 s) are shown in Fig. 14(c). It can be observed that although the FIR Q-filter produces almost zero steady-state tracking error after the first two periods, oscillations appear in the tracking results especially as the time elapses. These oscillations are induced by the high gain introduced by the FIR filter at high frequencies where model uncertainties are relevant [46]. This is the reason why the low-pass filter (16) is employed alternatively in the current research. As the sacrifice, the steady-state control error does not converge to zero exactly due to the phase-delay nature of this low-pass filter. Even so, the control error can be reduced by selecting a smaller time constant (\( T_q \)). With different values of \( T_q \), the tracking results are plotted in Fig. 14(d). It is found that too
small $T_q$ causes instability of the closed-loop system. Therefore, a tradeoff between tracking accuracy and robustness is required for the low-pass filter design. In this research, $T_q = 0.01$ s is selected.

To demonstrate the need of the $H_\infty$ controller design, it is also desirable to compare the control performance of RC alone with the $H_\infty + $RC method. However, without the designed $H_\infty$ controller $G_c$, the closed-loop system is unstable because the complementary sensitivity function $T$ in (5) has two poles locating outside the unit circle in $z$-plane. Hence, a stand-alone RC is not sufficient for the micropositioning system in the current research.

To further discover the necessity of combining the $H_\infty$ with RC, another simulation study is performed to track a 5-Hz triangular motion trajectory as described in Fig. 15(a). Similar to the treatment in [47], before using the triangular wave as the input, the original trajectory is filtered to remove high-frequency components by adopting a zero-phase digital filter with 30-Hz cutoff frequency. The filter is designed using the command “filtfilt” in Matlab. The tracking results of PID + RC and $H_\infty + $RC are shown in Fig. 15(a) and (b), respectively. Compared to PID results (the first cycle of tracking results), the PID + RC scheme reduces the control error by 70.2%, whereas the presented $H_\infty + $RC approach suppresses the tracking error more significantly by providing a 84.1% reduction.

V. EXPERIMENTAL VERIFICATION AND DISCUSSIONS

To verify the designed $H_\infty$ plus RC scheme, experimental studies are carried out to test the motion tracking performance of the XY piezostage system. The $H_\infty$ controller is constructed in Section IV-A and the repetitive controller parameters are set as $k_r = 1$ and $T_q = 0.01$. For the purpose of comparisons with simulation results, a sampling frequency of 2 kHz is assigned in the following experiments. Equations (1) to (16) represent the controller design process for the $H_\infty$ and repetitive control. They also illustrate in detail how to develop a combined control for a piezoactuated micropositioning system. The control algorithms are implemented with Matlab and Simulink software, and then downloaded to the DS1005 PPC board through ControlDesk.
A. Experimental Results

First, an experiment for a staircase input signal with 0.2-μm height is performed with the stand-alone $H_\infty$ controller, and the experimental results are shown in Fig. 16. It is clear to identify that the positioning resolution of the micropositioning stage is better than 0.2 μm.

Second, the single-axis motion tracking for a sinusoidal trajectory with peak-to-peak (p–p) amplitudes of 30 μm is experimentally tested. With the proposed controller, the tracking results are depicted in Fig. 17(a). From the first cycle (0.5–1.0 s), it can be deduced that the maximum p–p tracking error of the $H_\infty$ control is 1.939 μm, i.e., 6.5% of the motion range. Starting from the second cycle (1.0 s), the RC begins to take effect. After two cycles (1.5 s), the $H_\infty$ plus RC generates the maximum p–p error of 0.716 μm, which accounts for 2.4% of the motion range. Thus, in comparison with stand-alone $H_\infty$ control, the proposed controller substantially mitigates the tracking error by 63.1%.

Third, the motion tracking of a 5-Hz triangular-wave trajectory along the x-axis [see Fig. 18(a)] is carried out. The tracking errors are plotted in Fig. 18(b). With respect to the $H_\infty$ control results (the first cycle), the maximum p–p error has been reduced by the proposed control scheme from 1.412 μm to 0.884 μm, i.e., from 14.1% to 8.8% of the motion range, revealing an improvement of 37.4%. Comparing the experiment with simulation results for the sinusoidal (Figs. 14 and 17) and triangular (Figs. 15 and 18) motion tracking, we can observe that the experimental results agree well with the simulation output. Thus, the accuracy of the identified plant model (1) is verified.

Moreover, to disclose the two-axis cooperative tracking performance of the XY stage for 2-D motion, biaxial contouring tests are implemented. Other than the motion tracking error which reflects the difference between the desired and actual positions, contouring error is defined as the minimum distance between the actual position and desired trajectory along an orthogonal direction to the trajectory. For a circle of 5-μm radius, the tracking results of the X and Y axes are shown in Fig. 19, which are obtained with a contouring speed of 40 μm/s. For two speeds of 40 and 80 μm/s, the circular contouring results with and without the RC action are illustrated in Fig. 20. It is observed that $H_\infty$ plus RC generates better contouring results than the stand-alone $H_\infty$ control. For comparisons of the tracking results of $H_\infty$ and $H_\infty +$ RC (after two contouring periods),
the percentage maximum p–p contouring errors relative to the circle diameter are described in Fig. 21. The bar charts show that, for different feed speeds (40, 60, 80, and 100 µm/s), the $H_\infty$ plus RC strategy reduces the contouring errors by 28.1%, 33.7%, 28.6%, and 18.3%, respectively, as compared with the $H_\infty$ robust control without RC efforts.

**B. Discussions on Stage Performance**

The above experimental results demonstrate that the robust repetitive control can substantially reduce both single-axis tracking and biaxial contouring errors of the XY micropositioning stage under the influences of hysteresis and creep. Relatively, the $H_\infty$ plus RC exhibits more significant improvement on 1-D tracking performance than on 2-D contouring for the micropositioning system. In order to further improve the 2-D contouring accuracy of the system, the contouring control approach [48] may be employed in the future research.

The assembled XY stage has a dimension of $116 \times 116 \times 46$ mm$^3$. It is much more compact than a monolithic one fabricated from only a piece of material in [17], which achieves almost the same size of workspace with a much larger profile dimension of $250 \times 250 \times 15$ mm$^3$. Moreover, the proposed stage can be fabricated with more compact dimension if the PZTs with smaller size and leaf-spring flexures instead of the right-circular flexure hinges are adopted. Compared to a commercial XY piezostage produced by Physik Instrumente, e.g., model
P-541.2CD, which has a dimension of $150 \times 150 \times 16.5 \text{ mm}^3$, a closed-loop workspace $100 \times 100 \mu\text{m}^2$ with a closed-loop resolution of 0.2 nm, the developed stage has a more compact planar dimension with a larger workspace size while owns a much worse resolution. The limitation of the current piezostage mainly arises from the displacement sensors which have a limited resolution of 38 nm. The sensors are the bottle-neck in ultra-precision positioning stages. Use of high-performance sensors with subnanometer-level resolution is expected to significantly increase the system-resolution of the proposed stage. It is noticeable that, as the disadvantage of assembly of modules, the system dynamics and statics could change every time when the system was decomposed and then reassembled.

The current amplitudes of steady-state errors (see Figs. 17 and 18) indicate that the performance of the robust repetitive control may be improved, e.g., by designing a more suitable low-pass filter ($Q$), to further reduce the tracking error down to the positioning resolution. While this robust repetitive control study is preliminary and there is plenty of room for performance improvement, the enhancement of positioning precision for the XY micropositioning system over the sole $H_\infty$, RC control, and PID plus RC control elaborated by the conducted investigations demonstrates the effectiveness of the mechatronic synthesis and displays great potential for the future research. Considering the performances achieved by the piezostage system with the current hardware, the stage may be more suitable for the manipulation of microscopic objects such as biological cells with the size of tens of micrometers. In the future, experiments will be conducted to demonstrate the capability of the stage for such micromanipulation tasks.

VI. CONCLUSION

This paper has concentrated on the design and control of a piezo-driven XY parallel micropositioning stage with integrated parallel, decoupled, and stacked kinematics structure and sub-micron accuracy for micro-/nanomanipulation applications. A series of analytical, simulation, and experimental studies were undertaken to facilitate the system design, plant model identification, and controller verification. It is found that SISO control is sufficient for the decoupled XY stage and the $H_\infty$ plus repetitive control strategy enables substantial improvement on the positioning accuracy in both single-axis tracking and bi-axial contouring tasks with unmodeled hysteresis and creep effects. Moreover, a submicron accuracy is achieved by the micropositioning system, which verifies the effectiveness of the presented mechanism and control design processes. Future research will focus on a more sophisticated controller design with higher bandwidth. The proposed stage will be employed for nanopositioning using high-performance displacement sensors with subnanometer-level resolution. Moreover, the stage can also be miniaturized into micro- and mesoscales for pertinent micro-/nanomanipulation applications.

References


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